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THE CLASS OF FUZZY GENERALIZED CLOSED SETS OF TYPE s^θ AND ITS APPLICATIONS

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Abstract. In this paper a new type of generalized version of fuzzy closed set, viz., $fs^\theta g$-closed set is introduced and studied. Using this concept as a basic tool, here we introduce and study $fs^\theta g$-open and $fs^\theta g$-closed functions, the class of which are strictly larger than that of fuzzy open and fuzzy closed functions respectively. Afterwards, we introduce and study $fs^\theta g$-continuous and $fs^\theta g$-irresolute functions. Next we introduce $fs^\theta g$-regular, $fs^\theta g$-normal, $fs^\theta g$-compact and $fs^\theta g$-T_2-spaces and the applications of $fs^\theta g$-open and $fs^\theta g$-closed functions on these spaces are discussed.

1. Introduction

fg-closed set is introduced in [3, 4]. Afterwards, different types of generalized version of fuzzy closed sets are introduced and studied. In this context we have to mention [6, 7, 8, 11, 12, 13, 14, 16, 17, 18]. In [2] fuzzy semiopen set is introduced. Using this concept as a basic tool, here we introduce $fs^\theta g$-closed set, the class of which is an independent concept of fg-closed set. After introducing fuzzy m-structure in [1], fuzzy minimal space (m-space, for short) is introduced in [5]. However generalized version of different types of closed sets in fuzzy m-space are introduced and studied in [9, 10, 15].

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2. Preliminaries

Throughout this paper (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [20]. In [32], L.A. Zadeh introduced fuzzy set as follows: A fuzzy set A is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [32] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$ [32]. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [32] while AqB means A is quasi-coincident (q-coincident, for short) with B, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [30]. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy point x_t and a fuzzy set A, $x_t \in A$ means $A(x) \geq t$, i.e., $x_t \leq A$. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [20] and fuzzy interior [20] of A respectively. A fuzzy set A is called a fuzzy neighbourhood (fuzzy nbd, for short) of a fuzzy point x_α if there exists a fuzzy open set U in X such that $x_\alpha \in U \leq A$ [30]. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_α [30]. A fuzzy set A is called a fuzzy quasi neighbourhood (fuzzy q-nbd, for short) [30] of a fuzzy point x_α in an fts X if there is a fuzzy open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open q-nbd [30] of x_α. A fuzzy set A in X is called fuzzy semiopen [2] if $A \leq cl(intA)$. The complement of a fuzzy semiopen set is called fuzzy semiclosed [2]. The intersection (resp., union) of all fuzzy semiclosed (resp., fuzzy semiopen) sets containing (resp., contained in) a fuzzy set A is called fuzzy semiclosure [2] (resp., fuzzy semiinterior [2]) of A, to be denoted by $sclA$ (resp., $sintA$). The collection of all fuzzy semiopen (resp. fuzzy semiclosed) sets in an fts (X, τ) is denoted by $FSO(X, \tau)$ (resp., $FSC(X, \tau)$).

3. $fs^\theta g$-Closed Set

In this section $fs^\theta g$-closed set is introduced and studied. Some important properties of this newly defined set are discussed here .

Definition 3.1. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fs^\theta g$-closed set in X if $cl(sintA) \leq U$ whenever $A \leq U \in FSO(X)$.

The complement of this set is called $fs^\theta g$-open set in X.

The collection of all $fs^\theta g$-closed (resp., $fs^\theta g$-open) sets in an fts X is denoted by $Fs^\theta GC(X)$ (resp., $Fs^\theta GO(X)$).

Remark 3.2. Union and intersection of two $fs^\theta g$-closed sets may not be so, as it seen from the following example.

Example 3.3. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Here $FSO(X) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$. Now consider the fuzzy sets B and C defined by $B(a) = 0.5, B(b) = 0, C(a) = 0, C(b) = 0.5$. Clearly B and C are $fs^\theta g$-closed sets in (X, τ). Let $D = B \bigvee C$. Then $D(a) = D(b) = 0.5$. Now $D \leq D \in FSO(X)$. But $cl(sintD) = 1_X \setminus A \not\leq D \Rightarrow D$ is not $fs^\theta g$-closed set in X.
Again consider two fuzzy sets S and T defined by $S(a) = 0.6, S(b) = 0.5, T(a) = 0.5, T(b) = 0.7$. Then clearly $S, T \in Fs^\theta GC(X)$. Let $U = S \bigwedge T$. Then $U(a) = U(b) = 0.5$. Now $U \leq U \in FSO(X)$. But $cl(sintU) = 1_X \setminus A \not\leq U \Rightarrow U \notin Fs^\theta GC(X)$.

Note 3.4. So we can conclude that the set of all $fs^\theta g$-open sets in an fts (X, τ) does not form a fuzzy topology.

Theorem 3.5. Let (X, τ) be an fts and $A, B \in I^X$. If $A \leq B \leq cl(sintA)$ and A is $fs^\theta g$-closed set in X, then B is also $fs^\theta g$-closed set in X.

Proof. Let $U \in FSO(X)$ be such that $B \leq U$. Then by hypothesis, $A \leq B \leq U$. As A is $fs^\theta g$-closed set in X, $cl(sintA) \leq U$. Then $cl(sintA) \leq cl(sintB) \leq cl(sint(cl(sintA))) \leq cl(sintA) \leq U \Rightarrow B$ is $fs^\theta g$-closed set in X.

Theorem 3.6. Let (X, τ) be an fts and $A, B \in I^X$. If $int(sclA) \leq B \leq A$ and A is $fs^\theta g$-open set in X, then B is also $fs^\theta g$-open set in X.

Proof. $int(sclA) \leq B \leq A \Rightarrow 1_X \setminus A \leq 1_X \setminus B \leq 1_X \setminus int(sclA) = cl(sint(1_X \setminus A))$ where $1_X \setminus A$ is $fs^\theta g$-closed set in X. By Theorem 3.5, $1_X \setminus B$ is $fs^\theta g$-closed set in $X \Rightarrow B$ is $fs^\theta g$-open set in X.

Theorem 3.7. Let (X, τ) be an fts and $A \in I^X$. Then A is $fs^\theta g$-open set in X if and only if $K \leq int(sclA)$ whenever $K \leq A$ and K is fuzzy semiclosed set in (X, τ).

Proof. Let $A (\in I^X)$ be $fs^\theta g$-open set in X and $K \leq A$ where K is fuzzy semiclosed set in (X, τ). Then $1_X \setminus A \leq 1_X \setminus K$ where $1_X \setminus A$ is $fs^\theta g$-closed set in X and $1_X \setminus K$ is fuzzy semiopen set in (X, τ). By hypothesis, $cl(sint(1_X \setminus A)) \leq 1_X \setminus K \Rightarrow 1_X \setminus int(sclA) \leq 1_X \setminus K \Rightarrow K \leq int(sclA)$.

Conversely, let $K \leq int(sclA)$ whenever $K \leq A$, $K \in FSC(X)$. Then $1_X \setminus A \leq 1_X \setminus K$ where $1_X \setminus K \in FSO(X)$. By hypothesis, $1_X \setminus int(sclA) \leq 1_X \setminus K \Rightarrow cl(sint(1_X \setminus A)) \leq 1_X \setminus K \Rightarrow 1_X \setminus A$ is

$fs^\theta g$-closed set in $X \Rightarrow A$ is $fs^\theta g$-open set in X.

Theorem 3.8. Let (X, τ) be an fts and $A, B \in I^X$. If A is $fs^\theta g$-closed set in X and B is fuzzy semiclosed set in (X, τ) with $A \not{q} B$. Then $cl(sintA) \not{q} B$.

Proof. By hypothesis, $A \not{q} B \Rightarrow A \leq 1_X \setminus B \in FSO(X) \Rightarrow cl(sintA) \leq 1_X \setminus B \Rightarrow cl(sintA) \not{q} B$.

Remark 3.9. The converse of Theorem 3.8 may not be true, in general, as it seen from the following example.

Example 3.10. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B, C\}$ where $A(a) = 0.4, A(b) = 0.6, B(a) = 0.3, B(b) = 0.5, C(a) = 0.8, C(b) = 1$. Then (X, τ) is an fts. Consider the fuzzy set D defined by $D(a) = 0.4, D(b) = 0.5$. Then $D \leq D \in FSO(X)$. But $cl(sintD) = clD = 1_X \setminus B \not\leq D \Rightarrow D$ is not $fs^\theta g$-closed set in X. Again $D \not{q} (1_X \setminus C)$. Also $cl(sintD) = (1_X \setminus B) \not{q} (1_X \setminus C)$.

Now we recall the following definitions from [3, 4] for ready references.

Definition 3.11 [3, 4]. Let (X, τ) be an fts and $A \in I^X$. Then A is called fg-closed set if $clA \leq U$ whenever $A \leq U \in \tau$.

Remark 3.12. It is clear from next examples that fg-closed set and $fs^\theta g$-closed are independent concepts.

Example 3.13. $fs^\theta g$-closed sets don't have to be fg-closed. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.4, B(b) = 0.2$. Then (X, τ) is an fts. Then $FSO(X) = \{0_X, 1_X, U, V\}$ where $A \leq U \leq 1_X \setminus B, B \leq V \leq 1_X \setminus A$. Consider the fuzzy set C defined by $C(a) = C(b) = 0.5$. Then $C \leq A \in \tau$ (also $C \leq A \in FSO(X)$). Here $clC = 1_X \setminus B \not\leq A \Rightarrow C$ is not fg-closed set in X. But $cl(sintC) = 1_X \setminus A \leq A \Rightarrow C$ is $fs^\theta g$-closed set in X.

Example 3.14. fg-closed sets don't have to be $fs^\theta g$-closed.

Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ) is an fts. Now consider the fuzzy set B defined by $B(a) = B(b) = 0.5$. Then clearly B is fg-closed set but not $fs^\theta g$-closed set in X.

Definition 3.15. An fts (X, τ) is called $fT_{s^\theta}g$-space if every $fs^\theta g$-closed set in X is fuzzy closed set in X.

Now we introduce a new type of generalized version of neighbourhood system in an fts.

Definition 3.16. Let (X, τ) be an fts and x_α, a fuzzy point in X. A fuzzy set A is called $fs^\theta g$-neighbourhood ($fs^\theta g$-nbd, for short) of x_α, if there exists an $fs^\theta g$-open set U in X such that $x_\alpha \in U \leq A$. If, in addition, A is $fs^\theta g$-open set in X, then A is called an $fs^\theta g$-open nbd of x_α.

Definition 3.17. Let (X, τ) be an fts and x_α, a fuzzy point in X. A fuzzy set A is called $fs^\theta g$-quasi neighbourhood ($fs^\theta g$-q-nbd, for short) of x_α if there is an $fs^\theta g$-open set U in X such that $x_\alpha q U \leq A$. If, in addition, A is $fs^\theta g$-open set in X, then A is called an $fs^\theta g$-open q-nbd of x_α.

Note 3.18. (i) It is clear from definitions that every $fs^\theta g$-open set is an $fs^\theta g$-open nbd of each of its points. But every $fs^\theta g$-nbd of x_α may not be an $fs^\theta g$-open set containing x_α follows from the following example.
(ii) Also every fuzzy open nbd (resp., fuzzy open q-nbd) of a fuzzy point x_α is an $fs^\theta g$-open nbd (resp., $fs^\theta g$-open q-nbd) of x_α. But the converses are not necessarily true, in general, as it seen from the following example.

Example 3.19. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.3$. Then (X, τ) is an fts. Here $FSO(X) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus A$. Consider the fuzzy point $a_{0.4}$ and the fuzzy set D defined by $D(a) = 0.5, D(b) = 0.3$. Then clearly D is not an $fs^\theta g$-closed set in X and so $1_X \setminus D$ is not $fs^\theta g$-open set in X. Let us consider the fuzzy set C defined by $C(a) = 0.5, C(b) = 0.6$. Clearly C is $fs^\theta g$-closed set in X and so $1_X \setminus C$ is an $fs^\theta g$-open set in X with $1_X \setminus C \leq 1_X \setminus D$. Again $a_{0.4} \in 1_X \setminus C$. So $1_X \setminus D$ is an $fs^\theta g$-nbd of $a_{0.4}$, though it is not an $fs^\theta g$-open set in X.
Also consider the fuzzy set E defined by $E(a) = 0.3, E(b) = 0.7$ and the fuzzy point $a_{0.6}$. Then clearly E is an $fs^\theta g$-closed set in X and so $1_X \setminus E$ is $fs^\theta g$-open set in X containing $a_{0.6}$ and so $1_X \setminus E$ is $fs^\theta g$-open nbd of $a_{0.6}$. But there does not exist any open set U in X with $a_{0.6} \in U \leq 1_X \setminus E$. Hence $1_X \setminus E$ is not a fuzzy nbd of $a_{0.6}$. Next consider the fuzzy point $a_{0.4}$ and the fuzzy set E. Here $1_X \setminus E$ is $fs^\theta g$-open q-nbd of $a_{0.4}$. But there does not exist any fuzzy open set $U \leq 1_X \setminus E$ in X which is q-coincident with $a_{0.4}$. So $1_X \setminus E$ is not a fuzzy open q-nbd of $a_{0.4}$.

4. $fs^\theta g$-Open Function and $fs^\theta g$-Closed Function

In this section, we first introduce and study a new type of generalized version of fuzzy closure-like operator which is seen to be an idempotent operator. Then using this operator as a basic tool, two types of functions are introduced and characterized.

Definition 4.1. Let (X,τ) be an fts and $A \in I^X$. Then $fs^\theta g$-closure and $fs^\theta g$-interior of A, denoted by $fs^\theta gcl(A)$ and $fs^\theta gint(A)$, are defined as follows:

$fs^\theta gcl(A) = \bigwedge\{F : A \leq F, F$ is $fs^\theta g$-closed set in $X\}$,

$fs^\theta gint(A) = \bigvee\{G : G \leq A, G$ is $fs^\theta g$-open set in $X\}$.

Remark 4.2. It is clear from definition that for any $A \in I^X$, $A \leq fs^\theta gcl(A) \leq clA$. If A is $fs^\theta g$-closed set in an fts X, then $A = fs^\theta gcl(A)$. Similarly, $intA \leq fs^\theta gint(A) \leq A$. If A is $fs^\theta g$-open set in an fts X, then $A = fs^\theta gint(A)$. It follows from Remark 3.2 that $fs^\theta gcl(A)$ (resp., $fs^\theta gint(A)$) may not be $fs^\theta g$-closed (resp., $fs^\theta g$-open) set in an fts X.

Theorem 4.3. Let (X,τ) be an fts and $A \in I^X$. Then for a fuzzy point x_t in X, $x_t \in fs^\theta gcl(A)$ if and only if every $fs^\theta g$-open q-nbd U of x_t, UqA.

Proof. Let $x_t \in fs^\theta gcl(A)$ for any fuzzy set A in an fts X and F be any $fs^\theta g$-open q-nbd of x_t. Then $x_tqF \Rightarrow x_t \not\in 1_X \setminus F$ which is $fs^\theta g$-closed set in X. Then by Definition 4.1, $A \not\leq 1_X \setminus F \Rightarrow$ there exists $y \in X$ such that $A(y) > 1 - F(y) \Rightarrow AqF$.

Conversely, let for every $fs^\theta g$-open q-nbd F of x_t, FqA. If possible, let $x_t \not\in fs^\theta gcl(A)$. Then by Definition 4.1, there exists an $fs^\theta g$-closed set U in X with $A \leq U$, $x_t \not\in U$. Then $x_tq(1_X \setminus U)$ which being $fs^\theta g$-open set in X is $fs^\theta g$-open q-nbd of x_t. By assumption, $(1_X \setminus U)qA$ Hence $(1_X \setminus A)qA$, a contradiction.

Theorem 4.4. Let (X,τ) be an fts and $A, B \in I^X$. Then the following statements are true:

(i) $fs^\theta gcl(0_X) = 0_X$,
(ii) $fs^\theta gcl(1_X) = 1_X$,
(iii) $A \leq B \Rightarrow fs^\theta gcl(A) \leq fs^\theta gcl(B)$,
(iv) $fs^\theta gcl(A \bigvee B) = fs^\theta gcl(A) \bigvee fs^\theta gcl(B)$,
(v) $fs^\theta gcl(A \wedge B) \leq fs^\theta gcl(A) \wedge fs^\theta gcl(B)$, equality does not hold, in general, follows from Example 3.3,
(vi) $fs^\theta gcl(fs^\theta gcl(A)) = fs^\theta gcl(A)$.

Proof. (i), (ii) and (iii) are obvious.

(iv) From (iii), $fs^\theta gcl(A) \bigvee fs^\theta gcl(B) \leq fs^\theta gcl(A \bigvee B)$.
To prove the converse, let $x_\alpha \in fs^\theta gcl(A \bigvee B)$. Then by Theorem 4.3, for any $fs^\theta g$-open set U in X with $x_\alpha qU$, $Uq(A \bigvee B) \Rightarrow$ there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1 \Rightarrow$ either UqA or $UqB \Rightarrow$ either $x_\alpha \in fs^\theta gcl(A)$ or $x_\alpha \in fs^\theta gcl(B) \Rightarrow x_\alpha \in fs^\theta gcl(A) \bigvee fs^\theta gcl(B)$.
(v) Follows from (iii).

(vi) As $A \leq fs^\theta gcl(A)$, for any $A \in I^X$, $fs^\theta gcl(A) \leq fs^\theta gcl(fs^\theta gcl(A))$ (by (iii)).

Conversely, let $x_\alpha \in fs^\theta gcl(fs^\theta gcl(A)) = fs^\theta gcl(B)$ where $B = fs^\theta gcl(A)$. Let U be any $fs^\theta g$-open set in X with $x_\alpha qU$. Then UqB implies that there exists $y \in X$ such that $U(y) + B(y) > 1$. Let $B(y) = t$. Then $y_t qU$ and $y_t \in B = fs^\theta gcl(A)$. So UqA implies that $x_\alpha \in fs^\theta gcl(A)$. Hence $fs^\theta gcl(fs^\theta gcl(A)) \leq fs^\theta gcl(A)$. Consequently, $fs^\theta gcl(fs^\theta gcl(A)) = fs^\theta gcl(A)$.

Theorem 4.5. Let (X, τ) be an fts and $A \in I^X$. Then the following statements hold:
(i) $fs^\theta gcl(1_X \setminus A) = 1_X \setminus fs^\theta gint(A)$
(ii) $fs^\theta gint(1_X \setminus A) = 1_X \setminus fs^\theta gcl(A)$.

Proof (i). Let $x_t \in fs^\theta gcl(1_X \setminus A)$ for a fuzzy set A in an fts (X, τ). If possible, let $x_t \notin 1_X \setminus fs^\theta gint(A)$. Then $1 - (fs^\theta gint(A))(x) < t \Rightarrow [fs^\theta gint(A)](x) + t > 1 \Rightarrow fs^\theta gint(A)qx_t \Rightarrow$ there exists at least one $fs^\theta g$-open set $F \leq A$ with $x_t qF$ and so $x_t qA$. As $x_t \in fs^\theta gcl(1_X \setminus A), Fq(1_X \setminus A)$ Then $Aq(1_X \setminus A)$, a contradiction. Hence

$$fs^\theta gcl(1_X \setminus A) \leq 1_X \setminus fs^\theta gint(A)...(1)$$

Conversely, let $x_t \in 1_X \setminus fs^\theta gint(A)$. Then $1 - [(fs^\theta gint(A)](x) \geq t$. So $x_t \not q (fs^\theta gint(A))$. So $x_t \not q F$ for every $fs^\theta g$-open set F contained in A ... (2).
Let U be any $fs^\theta g$-closed set in X such that $1_X \setminus A \leq U$. Then $1_X \setminus U \leq A$. Now $1_X \setminus U$ is $fs^\theta g$-open set in X contained in A. By (2), $x_t \not q (1_X \setminus U)$ implies that $x_t \in U \Rightarrow x_t \in fs^\theta gcl(1_X \setminus A)$ and so

$$1_X \setminus fs^\theta gint(A) \leq fs^\theta gcl(1_X \setminus A)...(3).$$

Combining (1) and (3), (i) follows.
(ii) Putting $1_X \setminus A$ for A in (i), we get $fs^\theta gcl(A) = 1_X \setminus fs^\theta gint(1_X \setminus A)$. Hence $fs^\theta gint(1_X \setminus A) = 1_X \setminus fs^\theta gcl(A)$.

Let us now recall the following definition from [31] for ready references.

Definition 4.6 [31]. A function $f : X \to Y$ is called fuzzy open (resp., fuzzy closed) if $f(U)$ is fuzzy open (resp., fuzzy closed) set in Y for every fuzzy open (resp., fuzzy closed) set U in X.

Let us now introduce the following concept.

Definition 4.7. A function $h : X \to Y$ is called $fs^\theta g$-open function if $h(U)$ is $fs^\theta g$-open set in Y for every fuzzy open set U in X.

Remark 4.8. Since fuzzy open set is $fs^\theta g$-open set, we say that fuzzy open function is $fs^\theta g$-open function. But the converse need not be true, as it seen from the following example.

Example 4.9. $fs^\theta g$-open functions don't have to be fuzzy open. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_2) is $fs^\theta g$-open set in (X, τ_2), clearly i is $fs^\theta g$-open function. But $A \in \tau_1$, $i(A) = A \notin \tau_2 \Rightarrow i$ is not a fuzzy open function.

Theorem 4.10. For a bijective function $h : X \to Y$, the following statements are equivalent:
(i) h is $fs^\theta g$-open,
(ii) $h(intA) \leq fs^\theta gint(h(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy open set U in X containing x_α, there exists an $fs^\theta g$-open set V in Y containing $h(x_\alpha)$ such that $V \leq h(U)$.

Proof (i) $\Rightarrow$ (ii). Let $A \in I^X$. Then $intA$ is a fuzzy open set in X. By (i), $h(intA)$ is $fs^\theta g$-open set in Y. Since $h(intA) \leq h(A)$ and $fs^\theta gint(h(A))$ is the union of all $fs^\theta g$-open sets contained in $h(A)$, we have $h(intA) \leq fs^\theta gint(h(A))$.
(ii) $\Rightarrow$ (i). Let U be any fuzzy open set in X. Then $h(U) = h(intU) \leq fs^\theta gint(h(U))$ (by (ii)) $\Rightarrow h(U)$ is $fs^\theta g$-open set in $Y \Rightarrow h$ is $fs^\theta g$-open function.
(ii) $\Rightarrow$ (iii). Let x_α be a fuzzy point in X, and U, a fuzzy open set in X such that $x_\alpha \in U$. Then $h(x_\alpha) \in h(U) = h(intU) \leq fs^\theta gint(h(U))$ (by (ii)). Then $h(U)$ is $fs^\theta g$-open set in Y. Let $V = h(U)$. Then $h(x_\alpha) \in V$ and $V \leq h(U)$.
(iii) $\Rightarrow$ (i). Let U be any fuzzy open set in X and y_α, any fuzzy point in $h(U)$, i.e., $y_\alpha \in h(U)$. Then there exists unique $x \in X$ such that $h(x) = y$ (as h is bijective). Then $[h(U)](y) \geq \alpha \Rightarrow U(h^{-1}(y)) \geq \alpha \Rightarrow U(x) \geq \alpha \Rightarrow x_\alpha \in U$. By (iii), there exists $fs^\theta g$-open set V in Y such that $h(x_\alpha) \in V$ and $V \leq h(U)$. Then $h(x_\alpha) \in V = fs^\theta gint(V) \leq fs^\theta gint(h(U))$. Since y_α is taken arbitrarily and $h(U)$ is the union of all fuzzy points in $h(U)$, $h(U) \leq fs^\theta gint(f(U)) \Rightarrow h(U)$ is $fs^\theta g$-open set in $Y \Rightarrow h$ is an $fs^\theta g$-open function.

Theorem 4.11. If $h : X \to Y$ is $fs^\theta g$-open, bijective function, then the following statements are true:
(i) for each fuzzy point x_α in X and each fuzzy open q-nbd U of x_α in X, there exists an $fs^\theta g$-open q-nbd V of $h(x_\alpha)$ in Y such that $V \leq h(U)$,
(ii) $h^{-1}(fs^\theta gcl(B)) \leq cl(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy open q-nbd of x_α in X. Then $x_\alpha qU = intU \Rightarrow h(x_\alpha)qh(intU) \leq fs^\theta gint(h(U))$ (by Theorem 4.10 (i)$\Rightarrow$(ii)) implies that there exists at least one $fs^\theta g$-open q-nbd V of $h(x_\alpha)$ in Y with $V \leq h(U)$.
(ii) Let x_α be any fuzzy point in X such that $x_\alpha \notin cl(h^{-1}(B))$ for any $B \in I^Y$. Then there exists a fuzzy open q-nbd U of x_α in X such that $U \not{q} h^{-1}(B)$. Now

$$h(x_\alpha)qh(U)...(1)$$

where $h(U)$ is $fs^\theta g$-open set in Y. Now $h^{-1}(B) \leq 1_X \setminus U$ which is a fuzzy closed set in $X \Rightarrow B \leq h(1_X \setminus U)$ (as h is injective) $\leq 1_Y \setminus h(U)$. So $B \not{q} h(U)$. Let $V = 1_Y \setminus h(U)$. Then $B \leq V$ which is $fs^\theta g$-closed set in Y. We claim that $h(x_\alpha) \notin V$. If possible, let $h(x_\alpha) \in V = 1_Y \setminus h(U)$. Then $1 - [h(U)](h(x)) \geq \alpha \Rightarrow h(U) \not{q} h(x_\alpha)$, contradicting (1). So $h(x_\alpha) \notin V \Rightarrow h(x_\alpha) \notin fs^\theta gcl(B) \Rightarrow x_\alpha \notin h^{-1}(fs^\theta gcl(B)) \Rightarrow h^{-1}(fs^\theta gcl(B)) \leq cl(h^{-1}(B))$.

Theorem 4.12. An injective function $h : X \to Y$ is $fs^\theta g$-open if and only if for each $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$, there exists an $fs^\theta g$-closed set V in Y such that $B \leq V$ and $h^{-1}(V) \leq F$.

Proof. Let $B \in I^Y$ and F, a fuzzy closed set in X with $h^{-1}(B) \leq F$. Then $1_X \setminus h^{-1}(B) \geq 1_X \setminus F$ where $1_X \setminus F$ is a fuzzy open set in $X \Rightarrow h(1_X \setminus F) \leq h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus B$ (as h is injective) where $h(1_X \setminus F)$ is an $fs^\theta g$-open set in Y. Let $V = 1_Y \setminus h(1_X \setminus F)$. Then V is $fs^\theta g$-closed set in Y such that $B \leq V$. Now $h^{-1}(V) = h^{-1}(1_Y \setminus h(1_X \setminus F)) = 1_X \setminus h^{-1}(h(1_X \setminus F)) \leq F$.

Conversely, let F be a fuzzy open set in X. Then $1_X \setminus F$ is a fuzzy closed set in X. We have to show that $h(F)$ is an $fs^\theta g$-open set in Y. Now $h^{-1}(1_Y \setminus h(F)) \leq 1_X \setminus F$ (as h is injective). By assumption, there exists an $fs^\theta g$-closed set V in Y such that

$$1_Y \setminus h(F) \leq V...(1)$$

and $h^{-1}(V) \leq 1_X \setminus F$. Therefore, $F \leq 1_X \setminus h^{-1}(V)$ implies that

$$h(F) \leq h(1_X \setminus h^{-1}(V)) \leq 1_Y \setminus V...(2)$$

(as h is injective). Combining (1) and (2), $h(F) = 1_Y \setminus V$ which is an $fs^\theta g$-open set in Y. Hence h is $fs^\theta g$-open function.

Definition 4.13. A function $h : X \to Y$ is called $fs^\theta g$-closed function if $h(A)$ is $fs^\theta g$-closed set in Y for each fuzzy closed set A in X.

Remark 4.14. Since fuzzy closed set is $fs^\theta g$-closed set in an fts, we can conclude that every fuzzy closed function is $fs^\theta g$-closed function, but the converse may not be true as it follows from Example 4.9.

Here $1_X \setminus A \in \tau_1^c$, but $i(1_X \setminus A) = 1_X \setminus A \notin \tau_2^c \Rightarrow i$ is not a fuzzy closed function. But since every fuzzy set in (X, τ_2) is $fs^\theta g$-closed set in (X, τ_2), clearly i is $fs^\theta g$-closed function.

Theorem 4.15. A bijective function $h : X \to Y$ is $fs^\theta g$-closed function if and only if $fs^\theta gcl(h(A)) \leq h(clA)$, for all $A \in I^X$.

Proof. Let us suppose that $h : X \to Y$ be an $fs^\theta g$-closed function and $A \in I^X$. Then $h(cl(A))$ is $fs^\theta g$-closed set in Y. Since $h(A) \leq h(clA)$ and $fs^\theta gcl(h(A))$ is the intersection of all $fs^\theta g$-closed sets in Y containing $h(A)$, we have $fs^\theta gcl(h(A)) \leq h(clA)$.

Conversely, let for any $A \in I^X$, $fs^\theta gcl(h(A)) \leq h(clA)$. Let U be any fuzzy closed set in X. Then $h(U) = h(clU) \geq fs^\theta gcl(h(U)) \Rightarrow h(U)$ is an $fs^\theta g$-closed set in $Y \Rightarrow h$ is an $fs^\theta g$-closed function.

Theorem 4.16. If $h : X \to Y$ is an $fs^\theta g$-closed bijective function, then the following statements hold:
(i) for each fuzzy point x_α in X and each fuzzy closed set U in X with $x_\alpha \not q U$, there exists an $fs^\theta g$-closed set V in Y with $h(x_\alpha) \not q V$ such that $V \geq h(U)$,
(ii) $h^{-1}(fs^\theta gint(B)) \geq int(h^{-1}(B))$, for all $B \in I^Y$.

Proof (i). Let x_α be a fuzzy point in X and U be any fuzzy closed set in X with $x_\alpha \not q U = clU \Rightarrow h(x_\alpha) \not q h(clU) \geq fs^\theta gcl(h(U))$ (by Theorem 4.15) $\Rightarrow h(x_\alpha) \not q V$ for some $fs^\theta g$-closed set V in Y with $V \geq h(U)$.
(ii). Let $B \in I^Y$ and x_α be any fuzzy point in X such that $x_\alpha \in int(h^{-1}(B))$. Then there exists a fuzzy open set U in X with $U \leq h^{-1}(B)$ such that $x_\alpha \in U$. Then $1_X \setminus U \geq 1_X \setminus h^{-1}(B) \Rightarrow h(1_X \setminus U) \geq h(1_X \setminus h^{-1}(B))$ where $h(1_X \setminus U)$ is an $fs^\theta g$-closed set in Y. Let $V = 1_Y \setminus h(1_X \setminus U)$. Then V is an $fs^\theta g$-open set in Y and $V = 1_Y \setminus h(1_X \setminus U) \leq 1_Y \setminus h(1_X \setminus h^{-1}(B)) \leq 1_Y \setminus (1_Y \setminus B) = B$ (as h is injective). Now $U(x) \geq \alpha \Rightarrow x_\alpha \not q (1_X \setminus U) \Rightarrow h(x_\alpha) \not q h(1_X \setminus U) \Rightarrow h(x_\alpha) \leq 1_Y \setminus h(1_X \setminus U) = V \Rightarrow h(x_\alpha) \in V = fs^\theta gint(V) \leq fs^\theta gint(B) \Rightarrow x_\alpha \in h^{-1}(fs^\theta gint(B))$. Since x_α is taken arbitrarily, $int(h^{-1}(B)) \leq h^{-1}(fs^\theta gint(B))$, for all $B \in I^Y$.

Remark 4.17. Composition of two $fs^\theta g$-closed (resp., $fs^\theta g$-open) functions need not be so, as it seen from the following example.

Example 4.18. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = A(b) = 0.5$, $B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $fs^\theta g$-closed functions. Let $i_3 = i_2 \circ i_1 : (X, \tau_1) \to (X, \tau_3)$. We claim that i_3 is not $fs^\theta g$-closed function. Now $1_X \setminus A = A \in \tau_1^c$. $i_3(1_X \setminus A) = 1_X \setminus A \leq 1_X \setminus A \in FSO(X, \tau_3)$. But $cl_{\tau_3} sint_{\tau_3}(1_X \setminus A) =$

$1_X \setminus B \not\leq 1_X \setminus A \Rightarrow 1_X \setminus A$ is not an $fs^\theta g$-closed set in $(X, \tau_3) \Rightarrow i_3$ is not an $fs^\theta g$-closed function.

Similarly we can show that i_3 is not $fs^\theta g$-open function though i_1 and i_2 are so.

Theorem 4.19. If $h_1 : X \to Y$ is fuzzy closed (resp., fuzzy open) function and $h_2 : Y \to Z$ is $fs^\theta g$-closed (resp., $fs^\theta g$-open) function, then $h_2 \circ h_1 : X \to Z$ is $fs^\theta g$-closed (resp., $fs^\theta g$-open) function.

Proof. Obvious.

Now to establish the mutual relationship of $fs^\theta g$-closed function with the functions defined in [4], we have to recall the following definition first.

Definition 4.20 [4]. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called an fg-closed function if $h(A)$ is fg-closed set in Y for every $A \in \tau_1^c$.

Remark 4.21. fg-closed function and $fs^\theta g$-closed function are independent concepts follow from the following examples.

Example 4.22. $fs^\theta g$-closed functions don't have to be fg-closed.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.7, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus A \in \tau_1^c, i(1_X \setminus A) = 1_X \setminus A < B \in FSO(X, \tau_2)$. As $cl_{\tau_2} sint_{\tau_2}(1_X \setminus A) = 0_X \Rightarrow 1_X \setminus A$ is $fs^\theta g$-closed set in (X, τ_2) and hence i is an $fs^\theta g$-closed function. But $cl_{\tau_2}(1_X \setminus A) = 1_X \setminus B \not\leq B \Rightarrow 1_X \setminus A$ is not an fg-closed set in (X, τ_2). So i is not an fg-closed function.

Example 4.23. fg-closed functions don't have to be $fs^\theta g$-closed. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is an fg-closed function. Now $A \in \tau_1^c$, $i(A) = A \leq A \in FSO(X, \tau_2)$. But $cl_{\tau_2}(sint_{\tau_2} A) = 1_X \setminus B \not\leq A \Rightarrow A$ is not an $fs^\theta g$-closed set in (X, τ_2). Hence i is not an $fs^\theta g$-closed function.

5. $fs^\theta g$-Regular, $fs^\theta g$-Normal and $fs^\theta g$-Compact Spaces

In this section two new types of generalized version of fuzzy separation axioms are introduced and studied. Also a new type of generalized version of fuzzy compactness is introduced.

Definition 5.1. An fts (X, τ) is said to be $fs^\theta g$-regular space if for any fuzzy point x_t in X and each $fs^\theta g$-closed set F in X with $x_t \notin F$, there exist $U, V \in \tau$ such that $x_t \in U, F \leq V$ and $U \not{q} V$.

Theorem 5.2. In an fts (X, τ), the following statements are equivalent:
(i) X is $fs^\theta g$-regular,
(ii) for each fuzzy point x_t in X and any $fs^\theta g$-open q-nbd U of x_t, there exists $V \in \tau$ such that $x_t \in V$ and $clV \leq U$,
(iii) for each fuzzy point x_t in X and each $fs^\theta g$-closed set A of X with $x_t \notin A$, there exists $U \in \tau$ with $x_t \in U$ such that $clU \not q A$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any $fs^\theta g$-open q-nbd of x_t. Then $x_t q U \Rightarrow U(x) + t > 1 \Rightarrow x_t \notin 1_X \setminus U$ which is an $fs^\theta g$-closed set in X. By (i), there exist $V, W \in \tau$ such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not q W$. Then $V \leq 1_X \setminus W \Rightarrow clV \leq cl(1_X \setminus W) = 1_X \setminus W \leq U$.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A, an $fs^\theta g$-closed set in X with $x_t \notin A$. Then $A(x) < t \Rightarrow x_t q(1_X \setminus A)$ which being $fs^\theta g$-open set in X is $fs^\theta g$-open q-nbd of x_t. So by (ii), there exists $V \in \tau$ such that $x_t \in V$ and $clV \leq 1_X \setminus A$. Then $clV \not q A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any $fs^\theta g$-closed set in X with $x_t \notin F$. Then by (iii), there exists $U \in \tau$ such that $x_t \in U$ and $clU \not q F$. Then $F \leq 1_X \setminus clU$ (=V, say). So $V \in \tau$ and $V \not q U$ as $U \not q (1_X \setminus clU)$. Consequently, X is $fs^\theta g$-regular space.

Definition 5.3. An fts (X, τ) is called $fs^\theta g$-normal space if for each pair of $fs^\theta g$-closed sets A, B in X with $A \not q B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not q V$.

Theorem 5.4. An fts (X, τ) is $fs^\theta g$-normal space if and only if for every $fs^\theta g$-closed set F and $fs^\theta g$-open set G in X with $F \leq G$, there exists $H \in \tau$ such that $F \leq H \leq clH \leq G$.

Proof. Let X be $fs^\theta g$-normal space and let F be $fs^\theta g$-closed set and G be $fs^\theta g$-open set in X with $F \leq G$. Then $F \not q (1_X \setminus G)$ where $1_X \setminus G$ is $fs^\theta g$-closed set in X. By hypothesis, there exist $H, T \in \tau$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not q T$. Then $H \leq 1_X \setminus T \leq G$. Therefore, $F \leq H \leq clH \leq cl(1_X \setminus T) = 1_X \setminus T \leq G$.

Conversely, let A, B be two $fs^\theta g$-closed sets in X with $A \not q B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists $H \in \tau$ such that $A \leq H \leq clH \leq 1_X \setminus B \Rightarrow A \leq H, B \leq 1_X \setminus clH$ (=V, say). Then $V \in \tau$ and so $B \leq V$. Also as $H \not q (1_X \setminus clH)$, $H \not q V$. Consequently, X is $fs^\theta g$-normal space.

Let us now recall the following definitions from [20, 25] for ready references.

Definition 5.5. Let (X, τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\bigcup \mathcal{U} \geq A$ [25]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy regular open, $fs^\theta g$-open) in X, then $\mathcal{U}$ is called a fuzzy open [25] (resp., fuzzy regular open [2], $fs^\theta g$-open) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [20].

Definition 5.6. Let (X, τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A (resp., of X) is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [25]. If, in particular $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [20].

Definition 5.7. Let (X, τ) be an fts and $A \in I^X$. Then A is called fuzzy compact [20] (resp., fuzzy almost compact [21], fuzzy nearly compact [28]) set if every fuzzy open (resp., fuzzy open, fuzzy regular open) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup\limits_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [20] (resp., fuzzy almost compact [21], fuzzy nearly compact [22]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup\limits_{U \in \mathcal{U}_0} clU = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$).

Let us now introduce the following concept.

Definition 5.8. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fs^\theta g$-compact if every fuzzy cover $\mathcal{U}$ of A by $fs^\theta g$-open sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of $fs^\theta g$-compact space X.

Theorem 5.9. Every $fs^\theta g$-closed set in an $fs^\theta g$-compact space X is $fs^\theta g$-compact.

Proof. Let $A (\in I^X)$ be an $fs^\theta g$-closed set in an $fs^\theta g$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $fs^\theta g$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $fs^\theta g$-open sets of X. As X is $fs^\theta g$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Hence A is $fs^\theta g$-compact set.

Next we recall the following two definitions from [27, 26] for ready references.

Definition 5.10 [27]. An fts (X, τ) is called fuzzy regular space if for each fuzzy point x_t in X and each fuzzy closed set F in X with $x_t \notin F$, there exist $U, V \in \tau$ such that $x_t \in U$, $F \leq V$ and $U \not{q} V$.

Definition 5.11 [26]. An fts (X, τ) is called fuzzy normal space if for each pair of fuzzy closed sets A, B of X with $A \not{q} B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Remark 5.12. It is clear from above discussion that (i) $fs^\theta g$-regular (resp., $fs^\theta g$-normal, $fs^\theta g$-compact) space is fuzzy regular (resp., fuzzy normal, fuzzy compact) space, but the converses are not true, in general, follow from the following example.
(ii) In $fT_{s^\theta}g$-space, fuzzy regularity (resp., fuzzy normality, fuzzy compactness) implies $fs^\theta g$-regularity (resp., $fs^\theta g$-normality, fuzzy $fs^\theta g$-compactness).

Example 5.13. Let $X = \{a\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Clearly (X, τ) is fuzzy regular space, fuzzy normal space and fuzzy compact space. Here every fuzzy set is $fs^\theta g$-open as well as $fs^\theta g$-closed set in (X, τ). Consider the fuzzy point $a_{0.4}$ and the fuzzy set A defined by $A(a) = 0.3$. Then $a_{0.4} \notin A$ which is an $fs^\theta g$-closed set in X. But there do not exist $U, V \in \tau$ such that $a_{0.4} \in U, A \leq V$ and $U \not{q} V$. So (X, τ) is not $fs^\theta g$-regular space.
Similarly considering two fuzzy sets A, B defined by $A(a) = 0.3, B(a) = 0.1$. Then A and B are $fs^\theta g$-closed sets in X with $A \not{q} B$. But there do not exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$. So (X, τ) is not an $fs^\theta g$-normal space.
Again let $\mathcal{U} = \{U_n(a) : n \in N\}$ where $U_n(a) = \frac{n}{n+1}$, for all $n \in N$ of X. Then $\mathcal{U}$ is an $fs^\theta g$-open covering of X which has no finite subcovering. Hence (X, τ) is not an $fs^\theta g$-compact space.

6. $fs^\theta g$-Continuous and $fs^\theta g$-Irresolute Functions

In this section we first introduce two generalized version of functions and then establish the mutual relationships of these functions with the function defined in [4]. Afterwards the applications of these two functions on $fs^\theta g$-regular, $fs^\theta g$-normal and $fs^\theta g$-compact spaces are discussed here.

Now we first introduce the following concept.

Definition 6.1. A function $h : X \to Y$ is said to be $fs^\theta g$-continuous function if $h^{-1}(V)$ is $fs^\theta g$-closed set in X for every fuzzy closed set V in Y.

Theorem 6.2. Let $h : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:
(i) h is $fs^\theta g$-continuous function,
(ii) for each fuzzy point x_α in X and each fuzzy open nbd V of $h(x_\alpha)$ in Y, there exists an $fs^\theta g$-open nbd U of x_α in X such that $h(U) \leq V$,
(iii) $h(fs^\theta gcl(A)) \leq cl(h(A))$, for all $A \in I^X$,
(iv) $fs^\theta gcl(h^{-1}(B)) \leq h^{-1}(clB)$, for all $B \in I^Y$.

Proof (i) $\Rightarrow$ (ii). Let x_α be a fuzzy point in X and V, any fuzzy

open nbd of $h(x_\alpha)$ in Y. Then $x_\alpha \in h^{-1}(V)$ which is $fs^\theta g$-open set in X (by (i)). Let $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.
(ii) $\Rightarrow$ (i). Let A be any fuzzy open set in Y and x_α, a fuzzy point in X such that $x_\alpha \in h^{-1}(A)$. Then $h(x_\alpha) \in A$ where A is a fuzzy open nbd of $h(x_\alpha)$ in Y. By (ii), there exists an $fs^\theta g$-open nbd U of x_α in X such that $h(U) \leq A$. Then $x_\alpha \in U \leq h^{-1}(A) \Rightarrow x_\alpha \in U = fs^\theta gint(U) \leq fs^\theta gint(h^{-1}(A))$. Since x_α is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq fs^\theta gint(h^{-1}(A))$. So $h^{-1}(A)$ is an $fs^\theta g$-open set in X. Hence h is an $fs^\theta g$-continuous function.
(i) $\Rightarrow$ (iii). Let $A \in I^X$. Then $cl(h(A))$ is a fuzzy closed set in Y. By (i), $h^{-1}(cl(h(A)))$ is $fs^\theta g$-closed set in X. Now $A \leq h^{-1}(h(A)) \leq h^{-1}(cl(h(A)))$ and so $fs^\theta gcl(A) \leq fs^\theta gcl(h^{-1}(cl(h(A)))) = h^{-1}(cl(h(A)))$ and so $h(fs^\theta gcl(A)) \leq cl(h(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy closed set in Y. Put $U = h^{-1}(V)$. Then $U \in I^X$. By (iii), $h(fs^\theta gcl(U)) \leq cl(h(U)) = cl(h(h^{-1}(V))) \leq clV = V$ implies that $fs^\theta gcl(U) \leq h^{-1}(V) = U$ and so U is $fs^\theta g$-closed set in X Hence h is $fs^\theta g$-continuous function.
(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (iii), $h(fs^\theta gcl(A)) \leq cl(h(A))$. So $h(fs^\theta gcl(h^{-1}(B))) \leq cl(h(h^{-1}(B))) \leq clB$. Then $fs^\theta gcl(h^{-1}(B)) \leq h^{-1}(clB)$.
(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $h(A) \in I^Y$. By (iv), $fs^\theta gcl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A)))$. Then $fs^\theta gcl(A) \leq fs^\theta gcl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A))) \Rightarrow h(fs^\theta gcl(A)) \leq cl(h(A))$.

Remark 6.3. Composition of two $fs^\theta g$-continuous functions need not be so, as it seen from the following example.

Example 6.4. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, B\}$ $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.5, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Then clearly i_1 and i_2 are $fs^\theta g$-continuous functions. Now $A \in \tau_3^c$. So $(i_2 \circ i_1)^{-1}(A) = A < A \in FSO(X, \tau_1)$ (as $FSO(X, \tau_1) = \{0_X, 1_X, U\}$ where $B \leq U \leq 1_X \setminus B$). But $cl_{\tau_1} sint_{\tau_1}(A) = 1_X \setminus B \not\leq A$. So A is not an $fs^\theta g$-closed set in $(X, \tau_1) \Rightarrow i_2 \circ i_1$ is not an $fs^\theta g$-continuous function.

Let us now recall the following definition from [20] for ready references.

Definition 6.5 [20]. A function $h : X \to Y$ is called fuzzy continuous function if $h^{-1}(V)$ is fuzzy closed set in X for every fuzzy closed set V in Y.

Remark 6.6. Since every fuzzy closed set is $fs^\theta g$-closed set, it is clear that fuzzy continuous function is $fs^\theta g$-continuous function. But the converse is not necessarily true, follows from the following example.

Example 6.7. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $fs^\theta g$-closed set in (X, τ_1), clearly i is $fs^\theta g$-continuous function. But $A \in \tau_2^c$, $i^{-1}(A) = A \notin \tau_1^c \Rightarrow i$ is not fuzzy continuous function.

Theorem 6.8. If $h_1 : X \to Y$ is $fs^\theta g$-continuous function and $h_2 : Y \to Z$ is fuzzy continuous function, then $h_2 \circ h_1 : X \to Z$ is $fs^\theta g$-continuous function.

Proof. Obvious.

Theorem 6.9. If a bijective function $h : X \to Y$ is $fs^\theta g$-continuous, fuzzy open function from an $fs^\theta g$-regular space X onto an fts Y, then Y is fuzzy regular space.

Proof. Let y_α be a fuzzy point in Y and F, a fuzzy closed set in Y with $y_\alpha \notin F$. As h is bijective, there exists unique $x \in X$ such that $h(x) = y$. So $h(x_\alpha) \notin F \Rightarrow x_\alpha \notin h^{-1}(F)$ where $h^{-1}(F)$ is $fs^\theta g$-closed set in X (as h is an $fs^\theta g$-continuous function). As X is $fs^\theta g$-regular space, there exist $U, V \in \tau$ such that $x_\alpha \in U, h^{-1}(F) \leq V$ and $U \not{q} V$. Then $h(x_\alpha) \in h(U)$, $F = h(h^{-1}(F))$ (as h is bijective) $\leq h(V)$ and $h(U) \not{q} h(V)$ where $h(U)$ and $h(V)$ are fuzzy open sets in Y. (Indeed, $h(U) q h(V) \Rightarrow$ there exists $z \in Y$ such that $[h(U)](z) + [h(V)](z) > 1 \Rightarrow U(h^{-1}(z)) + V(h^{-1}(z)) > 1$ as h is bijective $\Rightarrow U q V$, a contradiction). Hence Y is a fuzzy regular space.

In a similar manner we can state the following theorems easily the proofs of which are same as that of Theorem 6.9.

Theorem 6.10. If a bijective function $h : X \to Y$ is $fs^\theta g$-continuous, fuzzy open function from an $fs^\theta g$-normal space X onto an fts Y, then Y is fuzzy normal space.

Theorem 6.11. If a bijective function $h : X \to Y$ is $fs^\theta g$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal), $fT_{s^\theta}g$-space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Definition 6.12. A function $h : X \to Y$ is called $fs^\theta g$-irresolute function if $h^{-1}(U)$ is an $fs^\theta g$-open set in X for every $fs^\theta g$-open set U in Y.

Theorem 6.13. A function $h : X \to Y$ is $fs^\theta g$-irresolute function if and only if for each fuzzy point x_α in X and each $fs^\theta g$-open nbd V in Y of $h(x_\alpha)$, there exists an $fs^\theta g$-open nbd U in X of x_α such that $h(U) \leq V$.

Proof. The proof is same as that of Theorem 6.2 (i)⇔(ii).

Now we state the following two theorems easily the proofs of which are similar to that of Theorem 6.9.

Theorem 6.14. If a bijective function $h : X \to Y$ is $fs^\theta g$-irresolute, fuzzy open function from an $fs^\theta g$-regular (resp., $fs^\theta g$-normal) space X onto an fts Y, then Y is $fs^\theta g$-regular (resp., $fs^\theta g$-normal) space.

Theorem 6.15. If a bijective function $h : X \to Y$ is $fs^\theta g$-irresolute, fuzzy open function from an $fs^\theta g$-regular (resp., $fs^\theta g$-normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.16. If a bijective function $h : X \to Y$ is $fs^\theta g$-irresolute, fuzzy open function from a fuzzy regular (resp., fuzzy normal), $fT_{s^\theta}g$-space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.17. Let $h : X \to Y$ be an $fs^\theta g$-continuous function from an fts X onto an fts Y and $A(\in I^X)$ be an $fs^\theta g$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \leq \bigcup\limits_{\alpha\in\Lambda} U_\alpha$ Then $A \leq h^{-1}(\bigcup\limits_{\alpha\in\Lambda} U_\alpha) = \bigcup\limits_{\alpha\in\Lambda} h^{-1}(U_\alpha)$. Then $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $fs^\theta g$-open sets of X as h is an $fs^\theta g$-continuous function. As A is $fs^\theta g$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \leq \bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)$. So $h(A) \leq h(\bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)) \leq \bigcup\limits_{\alpha\in\Lambda_0} U_\alpha$. Hence $h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Since fuzzy open set is $fs^\theta g$-open, we can state the following theorems easily the proofs of which are same as that of Theorem 6.17.

Theorem 6.18. Let $h : X \to Y$ be an $fs^\theta g$-irresolute function from an fts X onto an fts Y and $A(\in I^X)$ be an $fs^\theta g$-compact set in X. Then $h(A)$ is $fs^\theta g$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.19. Let $h : X \to Y$ be an $fs^\theta g$-continuous function from an $fs^\theta g$-compact space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.20. Let $h : X \to Y$ be an $fs^{\theta}g$-irresolute function from an $fs^{\theta}g$-compact space X onto an fts Y . Then Y is $fs^{\theta}g$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.21. Let $h : X \to Y$ be an $fs^{\theta}g$-continuous function from a fuzzy compact, $fT_{s^{\theta}}g$-space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.22. Let $h : X \to Y$ be an $fs^{\theta}g$-irresolute function from a fuzzy compact, $fT_{s^{\theta}}g$-space X onto an fts Y . Then Y is $fs^{\theta}g$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Remark 6.23. It is clear from definitions that (i) $fs^{\theta}g$-irresolute function is $fs^{\theta}g$-continuous, but the converse may not be true, as it seen from the following example.
Also (ii) fuzzy continuity and $fs^{\theta}g$-irresoluteness are independent concepts follow from the following examples.
(iii) Composition of two $fs^{\theta}g$-irresolute functions is also so.

Example 6.24. Fuzzy continuous functions, $fs^{\theta}g$-continuous functions don't have to be fgs^{θ}-irresolute.
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is $fs^{\theta}g$-continuous as well as fuzzy continuous function. Now every fuzzy set in (X, τ_2) is $fs^{\theta}g$-closed set in $(X.\tau_2)$. Consider the fuzzy set C defined by $C(a) = C(b) = 0.5$. Then C is $fs^{\theta}g$-closed set in (X, τ_2). Now $i^{-1}(C) = C < C \in FSO(X, \tau_1)$. But $cl_{\tau_1} sint_{\tau_1} C = 1_X \setminus A \not\leq C \Rightarrow C$ is not an $fs^{\theta}g$-closed set in $(X, \tau_1) \Rightarrow i$ is not an $fs^{\theta}g$-irresolute function.

Example 6.25. $fs^{\theta}g$-irresoluteness does not imply fuzzy continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since every fuzzy set in (X, τ_1) is $fs^{\theta}g$-closed set in (X, τ_1), clearly i is $fs^{\theta}g$-irresolute function. Also i is not fuzzy continuous function as $A \in \tau_2, i^{-1}(A) = A \notin \tau_1$.

Theorem 6.26. Let $h : X \to Y$ be an $fs^{\theta}g$-continuous function where Y is an $fT_{s^{\theta}}g$-space. Then h is $fs^{\theta}g$-irresolute function.

Proof. Obvious.

Theorem 6.27. If $h_1 : X \to Y$ is $fs^{\theta}g$-irresolute function and $h_2 : Y \to Z$ is $fs^{\theta}g$-continuous function, then $h_2 \circ h_1 : X \to Z$ is an

$fs^\theta g$-continuous function.

Proof. Obvious.

Let us first recall the definition of the function defined in [4].

Definition 6.28 [4]. Let $h : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then h is called fg-continuous function if $h^{-1}(V)$ is fg-closed set in X for every $V \in \tau_2^c$.

Remark 6.29. It is clear from definitions that fg-continuity and $fs^\theta s$-continuity are independent concepts follow from the following examples.

Example 6.30. $fs^\theta g$-continuity does not imply fg-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, B\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.7, B(a) = 0.5, B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \setminus A \in \tau_2^c$, $i^{-1}(1_X \setminus A) = 1_X \setminus A < B \in FSO(X, \tau_1)$ (also $B \in \tau_1$). Then $cl_{\tau_1} sint_{\tau_1}(1_X \setminus A) = 0_X < B$ implies that $1_X \setminus A$ is $fs^\theta g$-closed set in (X, τ_1) and so i is $fs^\theta g$-continuous function. But $cl_{\tau_1}(1_X \setminus A) = 1_X \setminus B \not\leq B$. Then $1_X \setminus A$ is not an fg-closed set in (X, τ_1). Hence i is not an fg-continuous function.

Example 6.31. fg-continuity does not imply $fs^\theta g$-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = B(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $B \in \tau_2^c$, $i^{-1}(B) = B \leq B \in FSO(X, \tau_1)$. But $cl_{\tau_1}(sint_{\tau_1} B) = 1_X \setminus A \not\leq B$. Then B is not $fs^\theta g$-closed set in (X, τ_1). So i is not an $fs^\theta g$-continuous function. Again 1_X is the only fuzzy open set in (X, τ_1) containing B and so i fg-continuous function.

Remark 6.32. Let $h : X \to Y$ be a function where X is an $fT_{s^\theta} g$-space. Then if h is an $fs^\theta g$-continuous function, then h is an fg-continuous function.

7. $fs^\theta g$-T_2-Space

A new type of fuzzy T_2-space is introduced in this section. Also a strong form of $fs^\theta g$-continuity is introduced and studied. Lastly the applications of this newly defined function and the functions defined earlier in this paper are established.

We first recall the definition and theorem from [27, 28] for ready references.

Definition 7.1 [27]. An fts (X, τ) is called fuzzy T_2-space if for any two distinct fuzzy points x_α and y_β; when $x \neq y$, there exist fuzzy open sets U_1, U_2, V_1, V_2 such that $x_\alpha \in U_1, y_\beta q V_1, U_1 \not{q} V_1$ and

$x_\alpha q U_2, y_\beta \in V_2, U_2 \not{q} V_2$; when $x = y$ and $\alpha < \beta$ (say), there exist fuzzy open sets U and V in X such that $x_\alpha \in U, y_\beta q V$ and $U \not{q} V$.

Theorem 7.2 [28]. An fts (X, τ) is fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Now we introduce the following concept.

Definition 7.3. An fts (X, τ) is called $fs^\theta g$-T_2-space if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist $fs^\theta g$-open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has an $fs^\theta g$-open nbd U and y_β has an $fs^\theta g$-open q-nbd V such that $U \not{q} V$.

Theorem 7.4. If an injective function $h : X \to Y$ is $fs^\theta g$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is $fs^\theta g$-T_2-space.

Proof. Let x_α and y_β be two distinct fuzzy points in X. Then $h(x_\alpha)$ ($= z_\alpha$, say) and $h(y_\beta)$ ($= w_\beta$, say) are two distinct fuzzy points in Y.

Case I. Suppose $x \neq y$. Then $z \neq w$. Since Y is fuzzy T_2-space, there exist fuzzy open sets U, V in Y such that $z_\alpha q U, w_\beta q V$ and $U \not{q} V$. As h is $fs^\theta g$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $fs^\theta g$-open sets in X with $x_\alpha q h^{-1}(U), y_\beta q h^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_\alpha q U \Rightarrow U(z) + \alpha > 1 \Rightarrow U(h(x)) + \alpha > 1 \Rightarrow [h^{-1}(U)](x) + \alpha > 1 \Rightarrow x_\alpha q h^{-1}(U)$. Again, $h^{-1}(U) q h^{-1}(V) \Rightarrow$ there exists $t \in X$ such that $[h^{-1}(U)](t) + [h^{-1}(V)](t) > 1 \Rightarrow U(h(t)) + V(h(t)) > 1 \Rightarrow U q V$, a contradiction].

Case II. Suppose $x = y$ and $\alpha < \beta$ (say). Then $z = w$ and $\alpha < \beta$. Since Y is fuzzy T_2-space, there exist a fuzzy open nbd U of x_α and a fuzzy open q-nbd V of w_β such that $U \not{q} V$. Then $U(z) \geq \alpha \Rightarrow [h^{-1}(U)](x) \geq \alpha \Rightarrow x_\alpha \in h^{-1}(U), y_\beta q h^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ where $h^{-1}(U)$ and $h^{-1}(V)$ are $fs^\theta g$-open sets in X as h is $fs^\theta g$-continuous function. Consequently, X is $fs^\theta g$-T_2-space.

Similarly we can state the following theorems easily the proofs of which are similar to that of Theorem 7.4.

Theorem 7.5. If a bijective function $h : X \to Y$ is $fs^\theta g$-irresolute function from an fts X onto an $fs^\theta g$-T_2-space Y, then X is $fs^\theta g$-T_2-space.

Theorem 7.6. If a bijective function $h : X \to Y$ is $fs^\theta g$-continuous function from an $fT_{s^\theta} g$-space X onto a fuzzy T_2-space Y, then X is fuzzy T_2-space.

Theorem 7.7. If a bijective function $h : X \to Y$ is $fs^\theta g$-irresolute function from an $fT_{s^\theta}g$-space X onto an $fs^\theta g$-T_2-space Y, then X is fuzzy T_2-space.

Theorem 7.8. If a bijective function $h : X \to Y$ is $fs^\theta g$-open function from a fuzzy T_2-space X onto an fts Y, then Y is $fs^\theta g$-T_2-space.

Theorem 7.9. If a bijective function $h : X \to Y$ is $fs^\theta g$-open function from a fuzzy T_2-space X onto an $fT_{s^\theta}g$-space Y, then Y is fuzzy T_2-space.

Now we introduce the strong form of $fs^\theta g$-continuous function.

Definition 7.10. A function $h : X \to Y$ is called strongly $fs^\theta g$-continuous function if $h^{-1}(V)$ is fuzzy closed set in X for every $fs^\theta g$-closed set V in Y.

Theorem 7.11. A function $h : X \to Y$ is strongly $fs^\theta g$-continuous function if and only if for each fuzzy point x_α in X and each $fs^\theta g$-open nbd V in Y of $h(x_\alpha)$, there exists a fuzzy open nbd U in X of x_α such that $h(U) \leq V$.

Proof. The proof is similar to that of Theorem 6.2 (i)$\Leftrightarrow$(ii).

Remark 7.12. (i) Composition of two strongly $fs^\theta g$-continuous functions is also so.
(ii) Strongly $fs^\theta g$-continuity implies fuzzy continuity, $fs^\theta g$-continuity and $fs^\theta g$-irresoluteness, but the reverse implications are not necessarily true, follow from the following examples.

Example 7.13. $fs^\theta g$-continuity, $fs^\theta g$-irresoluteness do not imply strongly $fs^\theta g$-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. As every fuzzy set in (X, τ_1) is $fs^\theta g$-closed set in (X, τ_1), clearly i is $fs^\theta g$-continuous as well as $fs^\theta g$-irresolute function. Now consider the fuzzy set B defined by $B(a) = B(b) = 0.4$. As $B \leq A \in FSO(X, \tau_2)$,we have $cl_{\tau_2}(sint_{\tau_2}B) = 0_X < A$, clearly B is $fs^\theta g$-closed set in (X, τ_2). But $i^{-1}(B) = B \notin \tau_1^c \Rightarrow i$ is not a strongly $fs^\theta g$-continuous function.

Example 7.14. Fuzzy continuity does not imply strongly $fs^\theta g$-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = A(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is fuzzy continuous function. Now every fuzzy set in (X, τ_2) is $fs^\theta g$-closed set in (X, τ_2). Consider the fuzzy set B defined by $B(a) = B(b) = 0.4$. Then B is $fs^\theta g$-closed set

in (X, τ_2). But $i^{-1}(B) = B \notin \tau_1^c \Rightarrow i$ is not a strongly $fs^\theta g$-continuous function.

Theorem 7.15. If $h_1 : X \to Y$ is strongly $fs^\theta g$-continuous function and $h_2 : Y \to Z$ is $fs^\theta g$-continuous function, then $h_2 \circ h_1 : X \to Z$ is fuzzy continuous function.

Proof. Obvious.

Now we can state the following theorems easily the proofs of which are similar to that of Theorem 6.9, Theorem 6.17 and Theorem 7.4.

Theorem 7.16. If a bijective function $h : X \to Y$ is strongly $fs^\theta g$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is $fs^\theta g$-regular (resp., $fs^\theta g$-normal) space.

Theorem 7.17. If a bijective function $h : X \to Y$ is strongly $fs^\theta g$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 7.18. If a bijective function $h : X \to Y$ is strongly $fs^\theta g$-continuous function from an fts X onto an $fs^\theta g$-T_2-space Y, then X is fuzzy T_2-space.

Theorem 7.19. If a bijective function $h : X \to Y$ is strongly $fs^\theta g$-continuous function from a fuzzy compact space X onto an fts Y, then Y is $fs^\theta g$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Remark 7.20. Clearly fuzzy T_2-space is $fs^\theta g$-T_2-space, but the converse is not necessarily true, follows from the following example.

Example 7.21. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Clearly (X, τ) is not a fuzzy T_2-space. Here every fuzzy set in (X, τ) is $fs^\theta g$-open set in (X, τ). Clearly X is $fs^\theta g$-T_2-space.

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