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Faculty of Sciences
Scientific Studies and Research
Series Mathematics and Informatics
Vol. 35 (2025), No. 2, 13-30

$fg\alpha^{\theta}$-CONTINUITY AND ITS APPLICATIONS IN FUZZY TOPOLOGICAL SPACES

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Abstract. This paper deals with different types of generalized version of fuzzy continuity, introducing the concept of $fg\alpha^{\theta}$-continuity, based on $fg\alpha^{\theta}$-closed sets and $fg\alpha^{\theta}$-open sets. Also, using $fg\alpha^{\theta}$-closed sets and $fg\alpha^{\theta}$-open sets, new types of fuzzy separation axioms and fuzzy compactness are studied. Some applications of $fg\alpha^{\theta}$-continuous functions are established.

1. **Introduction**

In [1], the notions of fuzzy regular open and fuzzy semiopen sets have been introduced, then in [10] fuzzy α-open set was defined. In [2, 3], fuzzy generalized versions of the notion of closed set have been introduced. Using fuzzy α-open sets as a basic tool, in [9] the concept of $fg\alpha^{\theta}$-closed set has been introduced and studied. Recall that the notions of fg-continuity and $fswg$-continuity have been introduced and studied in [3] and [8], respectively.

Keywords and phrases: Fuzzy regular open set, fuzzy semiopen set, fuzzy α-open set, $fg\alpha^{\theta}$-open set, $fg\alpha^{\theta}$-continuity, $fg\alpha^{\theta}$-irresoluteness, strongly $fg\alpha^{\theta}$-continuity, weakly $fg\alpha^{\theta}$-continuity, $fg\alpha^{\theta}$-regular space, $fg\alpha^{\theta}$-normal space.
(2020) Mathematics Subject Classification: 54A40, 03E72

In this paper, considering $fg\alpha^\theta$-closed sets [9] as a basic tool, we introduce the notion of $fg\alpha^\theta$-continuity. The class of $fg\alpha^\theta$-continuous functions is strictly larger than the class of fuzzy continuous functions and than the class of $fswg$-continuous functions. Also, we introduce the notion of $fg\alpha^\theta$-irresolute function and show that the class of $fg\alpha^\theta$-irresolute functions is strictly smaller than that of $fg\alpha^\theta$-continuous function. Moreover, we introduce $fg\alpha^\theta$-regular spaces, $fg\alpha^\theta$-normal spaces and $fg\alpha^\theta$-compact spaces, whose properties are strictly weaker than the property of a space to be fuzzy regular [16], fuzzy normal spaces [15] and fuzzy compact spaces [11].
Using the concept of $fg\alpha^\theta$-closed set, we introduce and study three different types of fuzzy continuous-like functions. Several applications of these types of functions on fuzzy regular, fuzzy normal, fuzzy compact and fuzzy T_2-spaces are obtained.

2. **Preliminaries**

Throughout this paper, by (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [11]. In [20], L.A. Zadeh introduced fuzzy set as follows: A fuzzy set A is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$.

The support [20] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$ [20].

For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [20] while AqB means A is quasi-coincident (q-coincident, for short) with B, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [18]. The negation of these two statements will be denoted by $A \not\leq B$ and $A\not{q}B$ respectively. For a fuzzy point x_t and a fuzzy set A, $x_t \in A$ means $A(x) \geq t$, i.e., $x_t \leq A$.

For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [11] and fuzzy interior [11] respectively. A fuzzy set A is called a fuzzy neighbourhood (fuzzy nbd, for short) of a fuzzy point x_α if there exists a fuzzy open set U in X such that $x_\alpha \in U \leq A$ [18]. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_α [18]. A fuzzy set A is called a fuzzy quasi neighbourhood (fuzzy q-nbd, for short) [18] of a fuzzy point x_α in a fts X if there is a fuzzy open set U in X

such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open q-nbd [18] of x_α.
A fuzzy set A in X is called
(1) fuzzy regular open [1] if $A = int(clA)$
(2) fuzzy semiopen [1] if $A \leq cl(intA)$
(3) fuzzy α-open [10] if $A \leq int(cl(intA))$.
The complement of a fuzzy regular open (resp., fuzzy α-open) set is called fuzzy regular closed [1] (resp., fuzzy α-closed [10]). The union (resp., intersection) of all fuzzy α-open (resp., fuzzy α-closed) sets contained in (resp., containing) a fuzzy set A is called fuzzy α-interior [10] (resp., fuzzy α-closure [10]) of A, to be denoted by $\alpha - intA$ (resp., $\alpha - clA$).

The collection of all fuzzy open (resp., fuzzy regular open, fuzzy semiopen, fuzzy α-open) sets in an fts (X, τ) is denoted by τ (resp., $FRO(X)$, $FSO(X)$, $F\alpha O(X)$). The collection of all fuzzy closed (resp., fuzzy regular closed, fuzzy α-closed) sets in an fts X is denoted by τ^c (resp., $FRC(X)$, $F\alpha C(X)$).

3. **Some properties of $fg\alpha^\theta$-closed sets**

In [9], we have introduced and studied the notion of $f\alpha^\theta g$-closed set. Now we recall some properties of $f\alpha^\theta g$-closed sets, which will be used in this paper.

Definition 3.1 [9]. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fg\alpha^\theta$-closed set in X if $cl(\alpha - intA) \leq U$ whenever $A \leq U \in \tau$.

The complement of $fg\alpha^\theta$-closed set is called $fg\alpha^\theta$-open set in X.
The collection of all $fg\alpha^\theta$-closed (resp., $fg\alpha^\theta$-open) sets in an fts X is denoted by $FG\alpha^\theta C(X)$ (resp., by $FG\alpha^\theta O(X)$).

Remark 3.2 [9]. The union and the intersection of two $fg\alpha^\theta$-closed sets may not be so.

Definition 3.3 [9]. Let (X, τ) be an fts and $A \in I^X$. The $fg\alpha^\theta$-closure and the $fg\alpha^\theta$-interior of A, denoted by $fg\alpha^\theta cl(A)$ and $fg\alpha^\theta int(A)$, are defined as follows:

$fg\alpha^\theta cl(A) = \bigwedge\{F : A \leq F, F \text{ is } fg\alpha^\theta\text{-closed set in } X\}$,
$fg\alpha^\theta int(A) = \bigvee\{G : G \leq A, G \text{ is } fg\alpha^\theta\text{-open set in } X\}$.

Definition 3.4 [9]. An fts (X, τ) is called $fT_{g\alpha^\theta}$-space if every $fg\alpha^\theta$-closed set in X is fuzzy closed set in X.

Definition 3.5 [9]. Let (X, τ) be an fts and x_t, a fuzzy point in X. A fuzzy set A is called $fg\alpha^\theta$-neighbourhood ($fg\alpha^\theta$-nbd, for short) of

x_t, if there exists a $fg\alpha^\theta$-open set U in X such that $x_t \in U \leq A$. If, in addition, A is $fg\alpha^\theta$-open set in X, then A is called an $fg\alpha^\theta$-open nbd of x_t.

Definition 3.6 [9]. Let (X, τ) be an fts and x_t be a fuzzy point in X. A fuzzy set A is called $fg\alpha^\theta$-quasi neighbourhood ($fg\alpha^\theta$-q-nbd, for short) of x_t if there is an $fg\alpha^\theta$-open set U in X such that $x_t q U \leq A$. If, in addition, A is $fg\alpha^\theta$-open set in X, then A is called an $fg\alpha^\theta$-open q-nbd of x_t.

Definition 3.7 [9]. An fts (X, τ) is called $fg\alpha^\theta$-T_2-Space if for any two distinct fuzzy points x_t and y_s in X; when $x \neq y$, there exist $fg\alpha^\theta$-open sets U, V in X such that $x_t q U$, $y_s q V$ and $U \not{q} V$; when $x = y$ and $t < s$ (say), x_t has an $fg\alpha^\theta$-open nbd U and y_s has an $fg\alpha^\theta$-open q-nbd V such that $U \not{q} V$.

Theorem 3.8 [17]. An fts (X, τ) is fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X; when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha q U$, $y_\beta q V$ and $U \not{q} V$; when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Now we recall the following definitions from [2, 3, 4] for ready references.

Definition 3.9. Let (X, τ) be an fts and $A \in I^X$. Then A is called
(i) fg-closed set [2, 3] if $clA \leq U$ whenever $A \leq U \in \tau$, the complement of an fg-closed set is called an fg-open set,
(ii) $fswg$-closed set [4] if $cl(intA) \leq U$ whenever $A \leq U \in FSO(X)$.

Definition 3.10 [19]. A function $f : X \to Y$ is called fuzzy open function if $f(U)$ is fuzzy open set in Y for every fuzzy open set U in X.

Definition 3.11. A function $h : X \to Y$ is called
(i) fuzzy continuous function [11] if $h^{-1}(U)$ is fuzzy closed set in X for all fuzzy closed set U in Y,
(ii) fg-continuous function [3] if $h^{-1}(U)$ is fg-closed set in X for all fuzzy closed set U in Y,
(iii) $fswg$-continuous function [8] if $h^{-1}(U)$ is $fswg$-closed set in X for all fuzzy closed set U in Y.

4. $fg\alpha^\theta$-CONTINUOUS FUNCTIONS

In this section, the concept of $fg\alpha^\theta$-continuous function is introduced and characterized. This concept is more general than fuzzy continuity, fg-continuity and $fswg$-continuity. Afterwards, we introduce property of $fg\alpha^\theta$-irresoluteness of a function, which implies the $fg\alpha^\theta$-continuity, but is independent of fuzzy continuity. Lastly, we introduce the property of strongly $fg\alpha^\theta$-continuity, which implies fuzzy continuity, $fg\alpha^\theta$-continuity and $fg\alpha^\theta$-irresoluteness.

Definition 4.1. A function $h : X \to Y$ is said to be $fg\alpha^\theta$-continuous if $h^{-1}(V)$ is $fg\alpha^\theta$-closed set in X for every fuzzy closed set V in Y.

Theorem 4.2. Let $h : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:
(i) h is $fg\alpha^\theta$-continuous function,
(ii) for each fuzzy point x_t in X and each fuzzy open nbd V of $h(x_t)$ in Y , there exists an $fg\alpha^\theta$-open nbd U of x_t in X such that $h(U) \leq V$,
(iii) $h(fg\alpha^\theta cl(A)) \leq cl(h(A))$, for all $A \in I^X$,
(iv) $fg\alpha^\theta cl(h^{-1}(B)) \leq h^{-1}(clB)$, for all $B \in I^Y$.

Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and V, any fuzzy open nbd of $h(x_t)$ in Y. Then $x_t \in h^{-1}(V)$ which is $fg\alpha^\theta$-open set in X (by (i)). Let $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.
(ii) $\Rightarrow$ (i). Let A be any fuzzy open set in Y and x_t, a fuzzy point in X such that $x_t \in h^{-1}(A)$. Then $h(x_t) \in A$ where A is a fuzzy open nbd of $h(x_t)$ in Y. By (ii), there exists an $fg\alpha^\theta$-open nbd U of x_t in X such that $h(U) \leq A$. Then $x_t \in U \leq h^{-1}(A)$ implies that $x_t \in U = fg\alpha^\theta int(U) \leq fg\alpha^\theta int(h^{-1}(A))$. Since x_t is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq fg\alpha^\theta int(h^{-1}(A))$. Then $h^{-1}(A)$ is an $fg\alpha^\theta$-open set in X. Hence h is an $fg\alpha^\theta$-continuous function.
(i) $\Rightarrow$ (iii). Let $A \in I^X$. Then $cl(h(A))$ is a fuzzy closed set in Y. By (i), $h^{-1}(cl(h(A)))$ is $fg\alpha^\theta$-closed set in X. Now $A \leq h^{-1}(h(A)) \leq h^{-1}(cl(h(A)))$ and so $fg\alpha^\theta cl(A) \leq fg\alpha^\theta cl(h^{-1}(cl(h(A)))) = h^{-1}(cl(h(A)))$ implies that $h(fg\alpha^\theta cl(A)) \leq cl(h(A))$.
(iii) $\Rightarrow$ (i). Let V be a fuzzy closed set in Y. Put $U = h^{-1}(V)$. Then $U \in I^X$. By (iii), $h(fg\alpha^\theta cl(U)) \leq cl(h(U)) = cl(h(h^{-1}(V))) \leq clV = V$. So $fg\alpha^\theta cl(U) \leq h^{-1}(V) = U$. Then U is an $fg\alpha^\theta$-closed set in X.

Consequently, h is an $fg\alpha^\theta$-continuous function.

(iii) $\Rightarrow$ (iv). Let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (iii), $h(fg\alpha^\theta cl(A)) \leq cl(h(A))$ implies that $h(fg\alpha^\theta cl(h^{-1}(B))) \leq cl(h(h^{-1}(B))) \leq clB$. So $fg\alpha^\theta cl(h^{-1}(B)) \leq h^{-1}(clB)$.

(iv) $\Rightarrow$ (iii). Let $A \in I^X$. Then $h(A) \in I^Y$. By (iv), $fg\alpha^\theta cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A)))$. Then $fg\alpha^\theta cl(A) \leq fg\alpha^\theta cl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A)))$. Hence $h(fg\alpha^\theta cl(A)) \leq cl(h(A))$.

Remark 4.3. The composition of two $fg\alpha^\theta$-continuous functions need not be so, as it is seen from the next example.

Example 4.4. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.6$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly i_1 and i_2 are $fg\alpha^\theta$-continuous functions. Let $i_3 = i_2 \circ i_1$. Then $i_3 : (X, \tau_1) \to (X, \tau_3)$. We claim that i_3 is not $fg\alpha^\theta$-continuous function. Now $1_X \setminus B \in \tau_3^c$. $i_3^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in \tau_1$. But $cl_{\tau_1}\alpha int_{\tau_1}(1_X \setminus B) = 1_X \setminus A \not\leq A$ which shows that $1_X \setminus B \notin FG\alpha^\theta C(X, \tau_1)$. As a result, i_3 is not $fg\alpha^\theta$-continuous function.

Remark 4.5. It is clear from definitions that
(i) Each of the properties of fuzzy continuity, fg-continuity and $fswg$-continuity implies $fg\alpha^\theta$-continuity. Indeed, fuzzy open sets, fg-open sets and $fswg$-open sets are $fg\alpha^\theta$-open sets [9]. But the reverse implications may not be true, in general, as follows from the next examples.

Example 4.6. An $fg\alpha^\theta$-continuous function is not necessarily fuzzy continuous, fg-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = B(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly, the function i is not fuzzy continuous. Now $1_X \setminus B = B \in \tau_2^c$, $i^{-1}(B) = B < A \in \tau_1$ and so $cl_{\tau_1}(\alpha - int_{\tau_1}B) = 0_X < A$. Then $B \in FG\alpha^\theta C(X, \tau_1)$. So i is $fg\alpha^\theta$-continuous function. Now $B \leq A \in \tau_1$. But $cl_{\tau_1}B = 1_X \not\leq A$ which implies that B is not fg-closed in (X, τ_1) and hence i is not an fg-continuous function.

Example 4.7. An $fg\alpha^\theta$-continuous function is not necessarily an $fswg$-continuous function
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = B(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's.

Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $1_X \backslash B = B \in \tau_2^c, i^{-1}(B) = B < 1_X$ in τ_1 only and so $cl_{\tau_1}(\alpha - int_{\tau_1} B) = 1_X \setminus A < 1_X$. Then $B \in FG\alpha^\theta C(X, \tau_1)$ implies that i is $fg\alpha^\theta$-continuous function. But $B \leq B \in FSO(X, \tau_1)$ and $cl_{\tau_1}(int_{\tau_1} B) = 1_X \setminus A \not\leq B$. Then B is not $fswg$-closed set in (X, τ_1). Consequently i is not $fswg$-continuous function.

Definition 4.8. A function $h : X \to Y$ is called $fg\alpha^\theta$-irresolute if $h^{-1}(U)$ is a $fg\alpha^\theta$-closed set in X for every $fg\alpha^\theta$-closed set U in Y.

Theorem 4.9. A function $h : X \to Y$ is $fg\alpha^\theta$-irresolute if and only if for each fuzzy point x_t in X and each $fg\alpha^\theta$-open nbd V in Y of $h(x_t)$, there exists a $fg\alpha^\theta$-open nbd U in X of x_t such that $h(U) \leq V$.
Proof. The proof is similar to that of Theorem 4.2 (i)⇔(ii).

Definition 4.10. A function $h : X \to Y$ is called strongly $fg\alpha^\theta$-continuous function if $h^{-1}(U)$ is a fuzzy closed set in X for all $fg\alpha^\theta$-closed set U in Y.

Theorem 4.11. A function $h : X \to Y$ is strongly $fg\alpha^\theta$-continuous if and only if for each fuzzy point x_t in X and each $fg\alpha^\theta$-open nbd V in Y of $h(x_t)$, there exists a fuzzy open nbd U in X of x_t such that $h(U) \leq V$.
Proof. The proof is similar to that of Theorem 4.2 (i)⇔(ii).

Definition 4.12. A function $h : X \to Y$ is called weakly $fg\alpha^\theta$-continuous function if $h^{-1}(U)$ is $fg\alpha^\theta$-closed set in X for all fuzzy regular closed set U in Y.

Theorem 4.13. A function $h : X \to Y$ is weakly $fg\alpha^\theta$-continuous function if and only if for each fuzzy point x_t in X and each $V \in FRO(Y)$ with $h(x_t) \in V$, there exists a $fg\alpha^\theta$-open nbd U in X of x_t such that $h(U) \leq V$.
Proof. The proof is similar to that of Theorem 4.2 (i)⇔(ii).

Remark 4.14. It is clear from definitions that
(i) As every fuzzy closed set is $fg\alpha^\theta$-closed set, strongly $fg\alpha^\theta$-continuity implies fuzzy continuity, $fg\alpha^\theta$-continuity and $fg\alpha^\theta$-irresoluteness, while $fg\alpha^\theta$-irresoluteness implies $fg\alpha^\theta$-continuity which implies weakly $fg\alpha^\theta$-continuity. The reverse implications are not necessarily true, as shown by the following examples.
(ii) fuzzy continuity and $fg\alpha^\theta$-irresoluteness are independent concepts,

as follows from the examples below.

(iii) The composition of two $fg\alpha^\theta$-irresolute (resp., strongly $fg\alpha^\theta$-continuous) functions is also so. But the composition of two weakly $fg\alpha^\theta$-continuous functions may not be so, as it is seen from the following example

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Example 4.15. Fuzzy continuity, $fg\alpha^\theta$-continuity may not imply $fg\alpha^\theta$-irresoluteness, strongly $fg\alpha^\theta$-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$ where $A(a) = 0.5, A(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is fuzzy continuous function as well as $fg\alpha^\theta$-continuous function. Now every fuzzy set in (X, τ_2) is $f\alpha^\theta g$-closed in (X, τ_2). Consider a fuzzy set C defined by $C(a) = 0.5, C(b) = 0.4$. Now $i^{-1}(C) = C \leq A \in \tau_1$. But $cl_{\tau_1}(\alpha - int_{\tau_1} C) = 1_X \setminus A \not\leq A$ implies that $C \notin FG\alpha^\theta C(X, \tau_1)$ and so i is not $fg\alpha^\theta$-irresolute function. Again, $C \notin \tau_1^c$ and so i is not strongly $fg\alpha^\theta$-continuous function.

Example 4.16. $fg\alpha^\theta$-irresoluteness does not imply neither fuzzy continuity, nor strongly $fg\alpha^\theta$-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$ where $A(a) = A(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly i is not fuzzy continuous function. Since every fuzzy set in (X, τ_1) is $fg\alpha^\theta$-closed set in (X, τ_1), i is clearly $fg\alpha^\theta$-irresolute function. Here $1_X \setminus A$ being fuzzy closed set in (X, τ_2) is $fg\alpha^\theta$-closed set in (X, τ_2). Now $i^{-1}(1_X \setminus A) = 1_X \setminus A \notin \tau_1^c$ which proves that the function i is not strongly $fg\alpha^\theta$-continuous.

Example 4.17. Weakly $fg\alpha^\theta$-continuity does not imply $fg\alpha^\theta$-continuity
Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since $FRC(X, \tau_2) = \{0_X, 1_X\}$, so clearly i is weakly $fg\alpha^\theta$-continuous function. Now $1_X \setminus B \in \tau_2^c, i^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in \tau_1$. But $cl_{\tau_1}(\alpha - int_{\tau_1}(1_X \setminus B)) = 1_X \setminus A \not\leq A$ implies that $1_X \setminus B$ is not $fg\alpha^\theta$-closed set in (X, τ_1). Hence i is not $fg\alpha^\theta$-continuous function.

Example 4.18. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) =$

0.4. Then $(X,\tau_1),(X,\tau_2)$ and (X,τ_3) are fts's. Consider two identity functions $i_1:(X,\tau_1)\to(X,\tau_2)$ and $i_2:(X,\tau_2)\to(X,\tau_3)$. Clearly i_1 and i_2 are weakly $fg\alpha^\theta$-continuous functions. Let $i_3=i_2\circ i_1$. Now $1_X\setminus B\in FRC(X,\tau_3), i_3^{-1}(1_X\setminus B)=1_X\setminus B\le A\in\tau_1$. But $cl_{\tau_1}(\alpha-int_{\tau_1}(1_X\setminus B))=1_X\not\le A$ implies that $1_X\setminus B$ is not $fg\alpha^\theta$-closed set in (X,τ_1). Hence i_3 is not weakly $fg\alpha^\theta$-continuous function.

Theorem 4.19. If $h_1:X\to Y$ is a strongly $fg\alpha^\theta$-continuous function and $h_2:Y\to Z$ is an $fg\alpha^\theta$-continuous function, then $h_2\circ h_1:X\to Z$ is fuzzy continuous function.
Proof. Obvious.

Note 4.20. Let $h:X\to Y$ be a $fg\alpha^\theta$-continuous function from a $fT_{g\alpha^\theta}$-space X onto an fts Y. Then h is fuzzy continuous, fg-continuous and $fswg$-continuous function.

5. $fg\alpha^\theta$-REGULAR, $fg\alpha^\theta$-NORMAL AND $fg\alpha^\theta$-COMPACT SPACES

In this section, two new types of separation axioms are introduced and studied. Also, a new type of compactness is introduced. Finally, the mutual relationships of these spaces with the spaces defined in [11, 12, 13, 15, 16] are established.

Definition 5.1. An fts (X,τ) is said to be $fg\alpha^\theta$-regular space if for any fuzzy point x_t in X and each $fg\alpha^\theta$-closed set F in X with $x_t\notin F$, there exist $U,V\in\tau$ such that $x_t\in U, F\le V$ and $U\not{q}V$.

Theorem 5.2. In an fts (X,τ), the following statements are equivalent:
(i) X is $fg\alpha^\theta$-regular,
(ii) for each fuzzy point x_t in X and any $fg\alpha^\theta$-open q-nbd U of x_t, there exists $V\in\tau$ such that $x_t\in V$ and $clV\le U$,
(iii) for each fuzzy point x_t in X and each $fg\alpha^\theta$-closed set A of X with $x_t\notin A$, there exists $U\in\tau$ with $x_t\in U$ such that $clU\not{q}A$.
Proof (i) $\Rightarrow$ (ii). Let x_t be a fuzzy point in X and U, any $fg\alpha^\theta$-open q-nbd of x_t. Then $x_tqU\Rightarrow U(x)+t>1\Rightarrow x_t\notin 1_X\setminus U$ which is an $fg\alpha^\theta$-closed set in X. By (i), there exist $V,W\in\tau$ such that $x_t\in V, 1_X\setminus U\le W$ and $V\not{q}W$. Then $V\le 1_X\setminus W\Rightarrow clV\le cl(1_X\setminus W)=1_X\setminus W\le U$.
(ii) $\Rightarrow$ (iii). Let x_t be a fuzzy point in X and A, an $fg\alpha^\theta$-closed set in X with $x_t\notin A$. Then $A(x)<t\Rightarrow x_tq(1_X\setminus A)$ which being an

$fg\alpha^\theta$-open set in X is $fg\alpha^\theta$-open q-nbd of x_t. So by (ii), there exists $V \in \tau$ such that $x_t \in V$ and $clV \leq 1_X \setminus A$. Then $clV \not{q} A$.
(iii) $\Rightarrow$ (i). Let x_t be a fuzzy point in X and F be any $fg\alpha^\theta$-closed set in X with $x_t \notin F$. Then by (iii), there exists $U \in \tau$ such that $x_t \in U$ and $clU \not{q} F$. Then $F \leq 1_X \setminus clU$ ($=V$, say). So $V \in \tau$ and $V \not{q} U$ as $U \not{q} (1_X \setminus clU)$. Consequently, X is $fg\alpha^\theta$-regular space.

Definition 5.3. An fts (X,τ) is called $fg\alpha^\theta$-normal space if for each pair of $fg\alpha^\theta$-closed sets A, B in X with $A \not{q} B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Theorem 5.4. An fts (X,τ) is $fg\alpha^\theta$-normal space if and only if for every $fg\alpha^\theta$-closed set F and $fg\alpha^\theta$-open set G in X with $F \leq G$, there exists $H \in \tau$ such that $F \leq H \leq clH \leq G$.

Proof. Let X be $fg\alpha^\theta$-normal space and let F be $fg\alpha^\theta$-closed set and G be $fg\alpha^\theta$-open set in X with $F \leq G$. Then $F \not{q} (1_X \setminus G)$ where $1_X \setminus G$ is $fg\alpha^\theta$-closed set in X. By hypothesis, there exist $H, T \in \tau$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not{q} T$. Then $H \leq 1_X \setminus T \leq G$. Therefore, $F \leq H \leq clH \leq cl(1_X \setminus T) = 1_X \setminus T \leq G$.

Conversely, let A, B be two $fg\alpha^\theta$-closed sets in X with $A \not{q} B$. Then $A \leq 1_X \setminus B$. By hypothesis, there exists $H \in \tau$ such that $A \leq H \leq clH \leq 1_X \setminus B \Rightarrow A \leq H, B \leq 1_X \setminus clH$ ($=V$, say). Then $V \in \tau$ and so $B \leq V$. Also as $H \not{q} (1_X \setminus clH)$, $H \not{q} V$. Consequently, X is $fg\alpha^\theta$-normal space.

Let us now recall the following definitions from [11, 14] for ready references.

Definition 5.5. Let (X,τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $\bigcup \mathcal{U} \geq A$ [14]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy regular open, $fg\alpha^\theta$-open) in X, then $\mathcal{U}$ is called a fuzzy open [14] (resp., fuzzy regular open [1], $fg\alpha^\theta$-open) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [11].

Definition 5.6. Let (X,τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [14]. If, in particular $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [11].

Definition 5.7. Let (X,τ) be an fts and $A \in I^X$. Then A is called fuzzy compact [11] (resp., fuzzy almost compact [12], fuzzy nearly compact [17]) set if every fuzzy open (resp., fuzzy open, fuzzy regular open) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [11] (resp., fuzzy almost compact [12], fuzzy nearly compact [13]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$).

Let us now introduce the concept of $fg\alpha^\theta$-compact space.

Definition 5.8. Let (X, τ) be an fts and $A \in I^X$. Then A is called $fg\alpha^\theta$-compact if every fuzzy cover $\mathcal{U}$ of A by $fg\alpha^\theta$-open sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of $fg\alpha^\theta$-compact space X.

Theorem 5.9. Every $fg\alpha^\theta$-closed set in an $fg\alpha^\theta$-compact space X is $fg\alpha^\theta$-compact.

Proof. Let $A (\in I^X)$ be an $fg\alpha^\theta$-closed set in an $fg\alpha^\theta$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $fg\alpha^\theta$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $fg\alpha^\theta$-open sets of X. As X is $fg\alpha^\theta$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Hence A is an $fg\alpha^\theta$-compact set.

Next we recall the following two definitions from [16, 15] for ready references.

Definition 5.10 [16]. An fts (X, τ) is called fuzzy regular space if for each fuzzy point x_t in X and each fuzzy closed set F in X with $x_t \notin F$, there exist $U, V \in \tau$ such that $x_t \in U$, $F \leq V$ and $U \not{q} V$.

Definition 5.11 [15]. An fts (X, τ) is called fuzzy normal space if for each pair of fuzzy closed sets A, B of X with $A \not{q} B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Remark 5.12. It is clear from above discussion that

(i) $fg\alpha^\theta$-regular (resp., $fg\alpha^\theta$-normal, $fg\alpha^\theta$-compact) space is fuzzy regular (resp., fuzzy normal, fuzzy compact) space, but the converses are not true, in general, as follows from the next example.

(ii) In an $fT_{g\alpha^\theta}$-space, fuzzy regularity (resp., fuzzy normality, fuzzy compactness) implies $fg\alpha^\theta$-regularity (resp., $fg\alpha^\theta$-normality, $fg\alpha^\theta$-compactness).

Example 5.13. Let $X = \{a\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is an fts. Clearly (X, τ) is fuzzy regular space, fuzzy normal space and fuzzy compact space. Here every fuzzy set is $fg\alpha^\theta$-open as well as $fg\alpha^\theta$-closed set in (X, τ). Consider the fuzzy point $a_{0.3}$ and the fuzzy set A defined by $A(a) = 0.2$. Then $a_{0.4} \notin A$ which is an $fg\alpha^\theta$-closed set in X. But there do not exist $U, V \in \tau$ such that $a_{0.3} \in U, A \leq V$ and $U\not{q}V$. So (X, τ) is not $fg\alpha^\theta$-regular space. Similarly, consider two fuzzy sets A, B defined by $A(a) = 0.3, B(a) = 0.4$. Then A and B are $fg\alpha^\theta$-closed sets in X with $A\not{q}B$. But there do not exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U\not{q}V$. So (X, τ) is not an $fg\alpha^\theta$-normal space. Let $\mathcal{U} = \{U_n(a) : n \in N\}$ where $U_n(a) = \frac{n}{n+1}$, for all $n \in N$ of X. Then $\mathcal{U}$ is an $fg\alpha^\theta$-open covering of X which has no finite subcovering. Hence (X, τ) is not an $fg\alpha^\theta$-compact space.

6. Applications

In this section several applications of the concepts defined in this paper are discussed.

Theorem 6.1. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-continuous, fuzzy open function from an $fg\alpha^\theta$-regular space X onto an fts Y, then Y is a fuzzy regular space.

Proof. Let y_t be a fuzzy point in Y and F, a fuzzy closed set in Y with $y_t \notin F$. As h is bijective, there exists unique $x \in X$ such that $h(x) = y$. So $h(x_t) \notin F \Rightarrow x_t \notin h^{-1}(F)$ where $h^{-1}(F)$ is $fg\alpha^\theta$-closed set in X (as h is an $fg\alpha^\theta$-continuous function). As X is $fg\alpha^\theta$-regular space, there exist fuzzy open sets U, V in X such that $x_t \in U, h^{-1}(F) \leq V$ and $U\not{q}V$. Then $h(x_t) \in h(U)$, $F = h(h^{-1}(F))$ (as h is bijective) $\leq h(V)$ and $h(U)\not{q}h(V)$ where $h(U)$ and $h(V)$ are fuzzy open sets in Y. (Indeed, $h(U)qh(V) \Rightarrow$ there exists $z \in Y$ such that $[h(U)](z) + [h(V)](z) > 1 \Rightarrow U(h^{-1}(z)) + V(h^{-1}(z)) > 1$ as h is bijective $\Rightarrow UqV$, a contradiction). Hence Y is a fuzzy regular space.

We can prove the following theorems using methods similar to that from the proof of Theorem 6.1.

Theorem 6.2. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-continuous, fuzzy open function from an $fg\alpha^\theta$-normal space X onto an fts Y, then Y is fuzzy normal space.

Theorem 6.3. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal), $fT_{g\alpha^\theta}$-space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.4. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-irresolute, fuzzy open function from an $fg\alpha^\theta$-regular (resp., $fg\alpha^\theta$-normal) space X onto an fts Y, then Y is $fg\alpha^\theta$-regular (resp., $fg\alpha^\theta$-normal) space.

Theorem 6.5. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-irresolute, fuzzy open function from an $fg\alpha^\theta$-regular (resp., $fg\alpha^\theta$-normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.6. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-irresolute, fuzzy open function from a fuzzy regular (resp., fuzzy normal), $fT_{g\alpha^\theta}$-space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.7. If a bijective function $h : X \to Y$ is strongly $fg\alpha^\theta$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is $fg\alpha^\theta$-regular (resp., $fg\alpha^\theta$-normal) space.

Theorem 6.8. If a bijective function $h : X \to Y$ is strongly $fg\alpha^\theta$-continuous, fuzzy open function from a fuzzy regular (resp., fuzzy normal) space X onto an fts Y, then Y is fuzzy regular (resp., fuzzy normal) space.

Theorem 6.9. Let $h : X \to Y$ be an $fg\alpha^\theta$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $fg\alpha^\theta$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \le \bigcup\limits_{\alpha\in\Lambda} U_\alpha \Rightarrow A \le h^{-1}(\bigcup\limits_{\alpha\in\Lambda} U_\alpha) = \bigcup\limits_{\alpha\in\Lambda} h^{-1}(U_\alpha)$. Then $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $fg\alpha^\theta$-open sets of X as h is an $fg\alpha^\theta$-continuous function. As A is an $fg\alpha^\theta$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \le \bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)$, which implies $h(A) \le h(\bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha) \le \bigcup\limits_{\alpha\in\Lambda_0} U_\alpha$, therefore, $h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y.

Since every fuzzy open set is $fg\alpha^\theta$-open, using the ideas from the proof of Theorem 6.9, the following theorems can be easily proved.

Theorem 6.10. Let $h : X \to Y$ be an $fg\alpha^\theta$-continuous function from an $fg\alpha^\theta$-compact space X onto an fts Y. Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.11. Let $h : X \to Y$ be an $fg\alpha^\theta$-irresolute function from X onto an fts Y and $A(\in I^X)$ be an $fg\alpha^\theta$-compact set in X. Then $h(A)$ is $fg\alpha^\theta$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.12. Let $h : X \to Y$ be an $fg\alpha^\theta$-irresolute function from an $fg\alpha^\theta$-compact space X onto an fts Y. Then Y is $fg\alpha^\theta$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) set in Y.

Theorem 6.13. Let $h : X \to Y$ be an $fg\alpha^\theta$-continuous function from a fuzzy compact, $fT_{g\alpha^\theta}$-space X onto an fts Y . Then Y is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.14. Let $h : X \to Y$ be an $fg\alpha^\theta$-irresolute function from a fuzzy compact, $fT_{g\alpha^\theta}$-space X onto an fts Y . Then Y is $fg\alpha^\theta$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.15. Let $h : X \to Y$ be a strongly $fg\alpha^\theta$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $fg\alpha^\theta$-compact set in X. Then $h(A)$ is a fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact, $fg\alpha^\theta$-compact) set in Y.

Theorem 6.16. Let $h : X \to Y$ be a strongly $fg\alpha^\theta$-continuous function from a fuzzy compact space X onto an fts Y. Then Y is $fg\alpha^\theta$-compact (resp., fuzzy compact, fuzzy almost compact, fuzzy nearly compact) space.

Theorem 6.17. Let $h : X \to Y$ be a weakly $fg\alpha^\theta$-continuous function from an fts X onto an fts Y and $A(\in I^X)$ be an $fg\alpha^\theta$-compact set in X. Then $h(A)$ is fuzzy nearly compact set in Y.

Theorem 6.18. Let $h : X \to Y$ be a weakly $fg\alpha^\theta$-continuous function from an $fg\alpha^\theta$-compact space X onto an fts Y. Then Y is fuzzy nearly compact space.

Theorem 6.19. Let $h : X \to Y$ be a weakly $fg\alpha^\theta$-continuous function from an $fT_{g\alpha^\theta}$-space X onto an fts Y and $A(\in I^X)$ be a fuzzy compact set in X. Then $h(A)$ is fuzzy nearly compact set in Y.

Theorem 6.20. Let $h : X \to Y$ be a weakly $fg\alpha^\theta$-continuous function from a fuzzy compact, $fT_{g\alpha^\theta}$-space X onto an fts Y. Then Y is fuzzy nearly compact space.

Theorem 6.21. If an injective function $h : X \to Y$ is $fg\alpha^\theta$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is $fg\alpha^\theta$-T_2-space.

Proof. Let x_t and y_s be two distinct fuzzy points in X. Then $h(x_t)$ $(= z_t$, say) and $h(y_s)$ $(= w_s$, say) are two distinct fuzzy points in Y.

Case I. Suppose that $x \neq y$. Then $z \neq w$. Since Y is fuzzy T_2-space, there exist fuzzy open sets U, V in Y such that $z_t qU, w_s qV$ and $U \not{q} V$. As h is $fg\alpha^\theta$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $fg\alpha^\theta$-open sets in X with $x_t qh^{-1}(U), y_s qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_t qU \Rightarrow U(z) + t > 1 \Rightarrow U(h(x)) + t > 1 \Rightarrow [h^{-1}(U)](x) + t > 1 \Rightarrow x_t qh^{-1}(U)$. Again, $h^{-1}(U)qh^{-1}(V) \Rightarrow$ there exists $p \in X$ such that $[h^{-1}(U)](p) + [h^{-1}(V)](p) > 1 \Rightarrow U(h(p)) + V(h(p)) > 1 \Rightarrow UqV$, a contradiction].

Case II. Suppose that $x = y$ and $t < s$ (say). Then $z = w$ and $t < s$. Since Y is fuzzy T_2-space, there exist a fuzzy open nbd U of x_t and a fuzzy open q-nbd V of w_s such that $U \not{q} V$. Then $U(z) \geq t \Rightarrow [h^{-1}(U)](x) \geq t \Rightarrow x_t \in h^{-1}(U), y_s qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ where $h^{-1}(U)$ and $h^{-1}(V)$ are $fg\alpha^\theta$-open sets in X as h is $fg\alpha^\theta$-continuous function. Consequently, X is $fg\alpha^\theta$-T_2-space.

We can prove the following theorems using methods similar to that from the proof of Theorem 6.21.

Theorem 6.22. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-irresolute function from an fts X onto an $fg\alpha^\theta$-T_2-space (resp., fuzzy T_2-space) Y, then X is an $fg\alpha^\theta$-T_2-space.

Theorem 6.23. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-continuous function from an $fT_{g\alpha^\theta}$-space X onto a fuzzy T_2-space Y, then X is a fuzzy T_2-space.

Theorem 6.24. If a bijective function $h : X \to Y$ is $fg\alpha^\theta$-irresolute function from an $fT_{g\alpha^\theta}$-space X onto an $fg\alpha^\theta$-T_2-space (resp., fuzzy T_2-space) Y, then X is a fuzzy T_2-space.

Theorem 6.25. If a bijective function $h : X \to Y$ is strongly $fg\alpha^\theta$-continuous function from an fts X onto an $fg\alpha^\theta$-T_2-space (resp., fuzzy T_2-space) Y, then X is a fuzzy T_2-space.

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