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ORIGINAL RESEARCH PAPER

MULTIFRACTAL HYDRODYNAMIC MODELING IN SCALE RELATIVITY WITH PERSPECTIVES FOR ENVIRONMENTAL, CHEMICAL, AND BIOTECHNOLOGICAL SYSTEMS

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Abstract: The study examines dynamic representations of complex systems in two multifractal scenarios: the Schrödinger multifractal scenario and the Madelung multifractal scenario, both being contextualized in Scale Relativity Theory. These scenarios exhibit the self-organization of complex systems and the emergence of new behavior. The numerical methods used in this study investigated changing patterns, symmetries, and vortex dynamics in various settings. These findings show that complex systems can exhibit self-similar (holographic) patterns, suggesting an intricate interplay between fractality and dynamics across different scales. It describes how these changes can dramatically change any existing viewpoint with reference to Scale Relativity Theory (SRT).

Keywords: *fractals, Scale Relativity Theory, Madelung scenario, non-differentiable geometry*

INTRODUCTION

Reconciling the geometric, smooth-manifold foundations of relativity with the intrinsically nonclassical structure of quantum dynamics remains a central theme of theoretical physics. Standard formulations of both special and general relativity presuppose a differentiable space-time manifold, so that velocities, accelerations, and curvature tensors are well-defined in the usual sense. Yet, from the quantum-mechanical standpoint, the "typical" trajectories that dominate the path-integral description are continuous but non-differentiable, with highly irregular (Brownian-like) behavior at arbitrarily fine resolution [1, 2]. This tension suggests that differentiability-often treated as an innocuous mathematical convenience-may encode a substantive physical assumption about the nature of space-time itself.

A complementary impetus comes from the broader study of scale-invariant and fractal structures in nature. Fractal geometry formalizes the observation that many physical sets and processes exhibit explicit dependence on observational resolution, with effective lengths, areas, or measures that diverge as the probing scale is refined [3]. In quantum mechanics, this idea becomes particularly concrete: the Hausdorff dimension of a quantum-mechanical path is two for a wide class of systems, reinforcing the view that quantum propagation is naturally associated with fractal-like curves rather than smooth worldlines [2]. Stochastic formulations, notably Nelson's stochastic mechanics, further underline the deep connection between Brownian kinematics and the Schrödinger equation, although they typically take the underlying space (-time) as given rather than deriving its geometry from first principles [4].

Scale Relativity Theory (SRT), developed by Laurent Nottale, addresses these converging themes by proposing that the principle of relativity be extended from transformations of motion to transformations of scale (i.e., changes in resolution) [5, 6]. In this view, "scale" is elevated to the status of a state variable of the reference system-analogous to velocity in motion relativity-so that physical laws should retain their form under changes of observational resolution. This postulate of scale covariance motivates a systematic generalization of space-time geometry: if one relinquishes the a priori assumption of differentiability, then continuous but non-differentiable coordinates generically imply a fractal (resolution-dependent) structure. SRT therefore frames quantum behaviour not as an additional set of axioms appended to classical mechanics, but as a manifestation of the deeper geometry of a non-differentiable space-time [6 – 8].

At the core of SRT is a geometric identification of physical trajectories with geodesics of a fractal space-time. In a differentiable manifold, geodesics form a well-behaved family parameterized by initial conditions. In a non-differentiable setting, however, geodesics become fractal and proliferate without bound, naturally inviting a statistical description [7, 9]. Moreover, non-differentiability breaks time-reversal symmetry at the infinitesimal level: the standard single-valued derivative with respect to time is replaced by a two-valued (forward/backward) structure. Nottale's construction encodes this two-valuedness using complex (and, in more general settings, quaternionic) formalisms, yielding a complex velocity field and a generalized derivative operator that is covariant under both coordinate and scale transformations [7, 9]. When applied to classical dynamical equations, this "scale-covariant" (or "quantum-covariant") derivative transforms them into Schrödinger-type equations under conditions corresponding to the critical fractal dimension $D_F = 2$, thereby reproducing standard non-relativistic quantum mechanics as a special case [7 – 9].

SRT also distinguishes between different “regimes” of scale transformation, in close analogy with the historical development from Galilean to Einsteinian relativity. In a first (often termed “Galilean”) approximation for scale laws, one obtains standard power-law scaling with constant fractal dimension, which mirrors inertial motion in the space of scales [5 – 8]. A fuller implementation of the relativity principle in scale space leads to a “special scale relativity” in which scale transformations take a Lorentz-like form in logarithmic variables. In this framework, the Planck length becomes an invariant minimal scale under dilations and contractions, playing for scale transformations a role analogous to the invariant speed of light in motion transformations [5, 7, 9]. This perspective aims to provide a geometric rationale for the emergence of fundamental cutoffs and transition scales (e.g., between classical and quantum domains), and it resonates with renormalization-group reasoning in quantum field theory, where physical quantities acquire explicit scale dependence under changes of resolution [5, 8].

Beyond foundational questions, SRT has been developed with a view toward applications across microphysics, astrophysics, and cosmology. Nottale and collaborators have argued that the same scale-covariant procedure that yields quantum-like equations at small scales can also arise effectively at very large time scales, especially in strongly chaotic systems where predictability horizons enforce a statistical description [9]. In this “macroscopic quantum-like” domain, quantization may appear as a structuring tendency of probability amplitudes constrained by boundary conditions and external fields, rather than as a microscopic postulate. Within astrophysical contexts, SRT has been applied to claims of quantized or preferential orbital structures and to broader problems of gravitational structuring [9]. In cosmology, the scale-relativistic extension has been used to motivate invariant maximal scales associated with the cosmological constant and to discuss implications for early-universe physics and large-scale dynamics [10].

A distinctive methodological feature of SRT is its attempt to derive quantum and scaling behavior from a modified differential geometry rather than importing probabilistic axioms ad hoc. In practical terms, this amounts to replacing the usual differential calculus on smooth manifolds with a scale-dependent calculus appropriate to non-differentiable curves, while preserving a generalized covariance principle [5, 7, 11]. The resulting framework is intended to be conservative in the sense that standard differentiable physics is recovered as a limiting or special case when fractal fluctuations become negligible above a transition scale [6, 9]. At the same time, it is ambitious in scope, proposing new interpretations of gauge transformations, coupling–scale relations, and the status of fundamental constants as emergent from scale dynamics [5, 8, 11].

In this paper, we aim to formulate and investigate a unified Scale Relativity Theory (SRT) framework for the dynamics of complex systems by developing two complementary multifractal representations-the Schrödinger scenario and the Madelung (hydrodynamic) scenario-and by using numerical analysis to characterize the resulting nonlinear evolution. Specifically, we examine how scale-dependent (non-differentiable) geometry generates self-organization, pattern formation, symmetry properties, and vortex-like structures across resolutions, and we assess the extent to which these emergent structures display self-similar (holographic-type) behavior that links local multifractal fluctuations to global dynamical organization.

DYNAMIC DESCRIPTIONS OF COMPLEX SYSTEMS BY MEANS OF TWO SCENARIOS

In the following, let us describe any complex system dynamics [12] by means of SRT [5, 11]. Subsequently, in accordance with SRT, two scenarios become compatible in the description of such dynamics [13]:

i) a Schrödinger multifractal scenario described by the differential equation (for details see [13, 14]):

$$\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}-2\right]}\partial^l\partial_l\psi + i\lambda(dt)^{\left[\frac{2}{f(\alpha)}-1\right]}\partial_t\psi = 0 \tag{1}$$

where

$$\partial_t = \frac{\partial}{\partial t}, \partial_l = \frac{\partial}{\partial x^l}, \partial^l\partial_l = \frac{\partial^2}{\partial x_l^2} \tag{2 a-c}$$

In Equations (1) and (2a-c), the variables are defined as follows: x_l represents the multifractal (non-differentiable) spatial coordinate, t denotes the non-multifractal (differentiable) temporal coordinate, λ signifies the multifractal – non-multifractal transition coefficient, $f(\alpha)$ indicates the singularity spectrum of order α, $\alpha \equiv \alpha(D_F)$ with D_F the motion curves' fractal dimension, dt is the scale resolution, and ψ is the state function [5, 15].

It is observed that, for, $\lambda = \hbar/2m_0$ (where $\hbar$ denotes the reduced Planck constant and m_0 is the microparticle's rest mass) and $D_F = 2$ [5,11], the monofractal dynamics of complex systems can be delineated by Peano-type curves. Subsequently, Equation (1) is reduced to Schrödinger's differential equation from Quantum Mechanics;

ii) a Madelung multifractal scenario characterized by a multifractal hydrodynamics differential equations system [13, 14]:

$$\begin{gathered}\partial_t V_D^i + \left(V_D^l\partial_l\right)V_D^i = -\partial^i Q \\ \partial_t\rho + \partial^i\left(\rho V_D^i\right) = 0\end{gathered} \tag{3 a, b}$$

with

$$\begin{gathered}\hat{V}^l = V_D^l - iV_F^l \\ Q = -2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}-2\right]}\frac{\partial^l\partial_l\sqrt{\rho}}{\sqrt{\rho}} = -\frac{V_F^l V_{Fl}}{2} - \lambda(dt)^{\left[\frac{2}{f(\alpha)}-1\right]}\partial_i V_F^i \\ V_D^i = 2\lambda(dt)^{\left[\frac{4}{f(\alpha)}-2\right]}\partial^i\phi, \qquad V_F^i = \lambda(dt)^{\left[\frac{2}{f(\alpha)}-1\right]}\partial^i\phi \\ \psi = \sqrt{\rho}e^{i\phi}, \bar{\psi} = \sqrt{\rho}e^{-i\phi},\ \rho = \psi\bar{\psi},\end{gathered} \tag{4 a-g}$$

where $\hat{V}^l$ represents the global velocity, V_D^l denotes the differentiable velocity, V_F^l signifies the non-differentiable velocity, Q indicates the specific multifractal potential, $\sqrt{\rho}$ refers to the amplitude and ϕ represents the phase of the state function. This system of equations pertains to two conservation principles: a specific multifractal momentum conservation law denotes as Equation (3a), and a multifractal states density conservation

law represented as Equation (3b). The multifractal potential in the Madelung framework may substantially impact in force generation. This influence originates from the intricate and nonlinear characteristics of the multifractal potential and its interaction with the dynamical structures linked to a quantum system delineated by Madelung formalism.
Regardless of the circumstance examined, employing $f(\alpha)$ offers the following benefits [11 – 14]:

i) the presence of a prevailing fractal dimension in any complex system dynamics enables the identification of this pattern a universal pattern. The explanation of this pattern corresponds with the distinctive global structural and functional traits to monofractal dynamics. The monofractal specific potential (4 b) can produce a monofractal viscosity stress tensor type:

$$\hat{\sigma}_{il} = m_0 D^2 (dt)^{(4/D_F)-2}[\nabla_i\rho\nabla_l\rho - (\nabla_i\rho\nabla_l\rho/\rho)] = \qquad (5\text{ a, b})$$
$$= \eta\left(\frac{\partial u_i}{\partial x_l} + \frac{\partial u_l}{\partial x_i}\right), \eta = \frac{1}{2} m_0 \rho D (dt)^{(2/D_F)-1}$$

of which divergence is equal to the usual force density associated with Q:

$$\nabla_i \hat{\sigma}_{il} = -\rho\nabla_l Q \qquad (6)$$

ii) the presence of a fractal dimensions "set" in any complex system dynamics enables the identification of zonal patterns. The explanation of these patterns corresponds with local structural and functional traits specific to multifractal dynamics.

iii) the singularity spectrum of order α facilitates the identification of universality classes within any complex system dynamics, even when the corresponding attractors display disparate properties.

NON-LINEAR BEHAVIORS

In the following, using equations (3 a, b) in a plane symmetry, we analyze, for example, the dynamics of the complex fluid, assuming that the fractal fluid is of isentropic-type. Thus, we can choose the tensor (5 a, b) in the form $\sigma_{il} = \rho c^2 \delta_{il}$ (5a), where ρ is the density, c is the specific velocity and δ_{il} is the Kronecker symbol. The presence of an external perturbation is specified by adequate initial and boundary conditions (e.g. spatio-temporal Gaussian). In this situation, let us introduce the normalized coordinates.

$$\omega t = \tau,\ kx = \xi, ky = \eta, \frac{V_{Dx}k}{\omega} = V_{\xi,} \frac{V_{Dy}k}{\omega} = V_{\eta,} \frac{\rho}{\rho_0} = N \qquad (7\text{ a-f})$$

where ω, k and ρ_0 are critical parameters of the complex fluid.
Then, equations (3 a, b) become:

$$\frac{\partial}{\partial\tau}(NV_\xi) + \frac{\partial}{\partial\xi}(NV_\xi^2) + \frac{\partial}{\partial\eta}(NV_\xi V_\eta) = -N^{-1}\frac{\partial N}{\partial\xi} \qquad (8\text{ a-c})$$

$$\frac{\partial}{\partial \tau}\left(NV_{\eta}\right)+\frac{\partial}{\partial \xi}\left(NV_{\xi}V_{\eta}\right)+\frac{\partial}{\partial \eta}\left(NV_{\eta}^{2}\right)=-N^{-1}\frac{\partial N}{\partial \eta}$$

$$\frac{\partial N}{\partial \tau}+\frac{\partial}{\partial \xi}\left(NV_{\xi}\right)+\frac{\partial}{\partial \eta}\left(NV_{\eta}\right)=0$$

For numerical integration we shall impose the initial conditions:

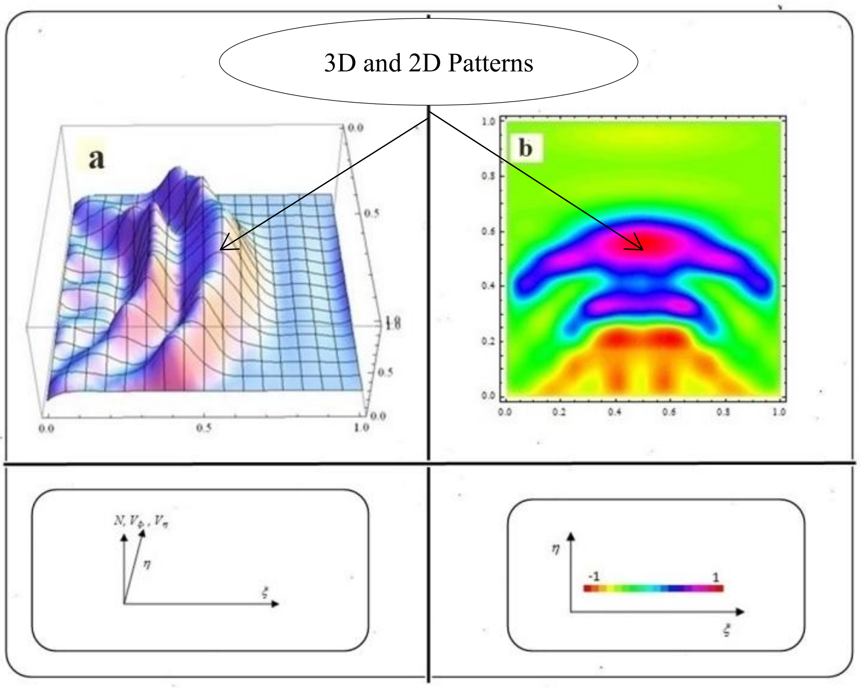
$$\begin{gathered} V_{\xi}(0,\xi,\eta)=0 \\ V_{\eta}(0,\xi,\eta)=0 \\ N(0,\xi,\eta)=\frac{1}{5} \\ 1\leq \xi \leq 2 \\ 0\leq \eta \leq 1 \end{gathered} \tag{9 a-e}$$

as well as the boundary conditions:

$$\begin{aligned} &V_{\xi}(\tau,1,\eta)=V_{\xi}(\tau,2,\eta)=0,\quad V_{\eta}(\tau,1,\eta)=V_{\eta}(\tau,2,\eta)=0 \\ &V_{\xi}(\tau,\xi,0)=V_{\xi}(\tau,\xi,1)=0,\quad V_{\eta}(\tau,\xi,0)=V_{\eta}(\tau,\xi,1)=0 \\ &N(\tau,1,\eta)=N(\tau,2,\eta)=\frac{1}{6} \\ &N(\tau,\xi,0)=\frac{1}{12}exp\left[-\left(\frac{\tau-\frac{1}{6}}{\frac{1}{6}}\right)^{2}\right]exp\left[-\left(\frac{\xi-\frac{6}{4}}{\frac{1}{6}}\right)^{2}\right] \\ &N(\tau,\xi,1)=\frac{1}{6} \end{aligned} \tag{10 a-g}$$

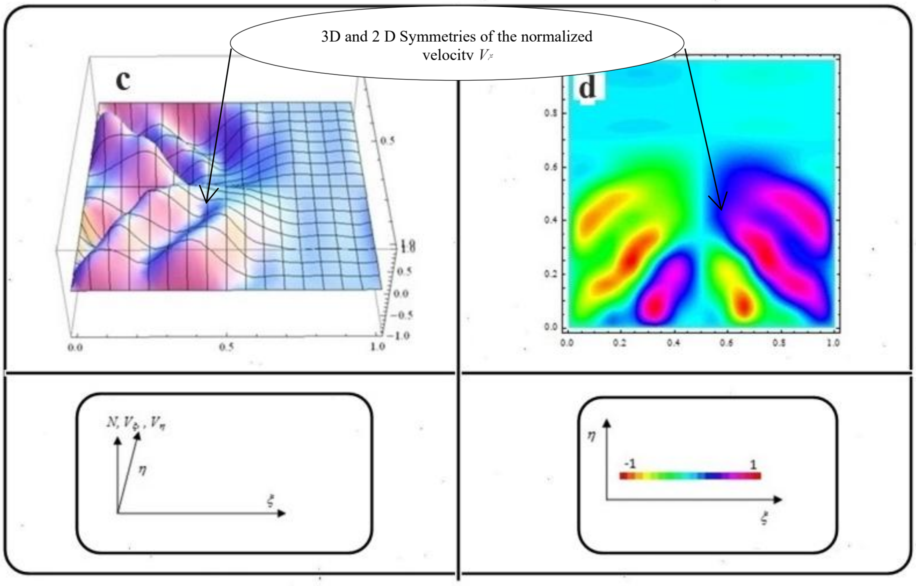
By using the finite differences method [16], the system (8 a-c) with the initial conditions (9 a-e) and the boundary ones (10a-g) was numerically resolved.

Figures 1 a, b; 2 c, d; 3 e, f presents the 3-D (three-dimensional) dependences of the normalized density N, normalized velocitys, V_{ξ} and V_{η}, on the normalized coordinates, ξ and η for the normalized time $\tau = 0.7$ (a, c, e) and 2-D (two dimensional) contour of the normalized density N, normalized velocitys, V_{ξ} and V_{η}, for the same normalized time (b, d, f).



3D and 2D Patterns

Figure 1. *a, b three dimensional (3D) and countour plot (2D) of the normalized density N, on the normalized coordinates, ξ and η for the normalized time τ =0.7*



3D and 2 D Symmetries of the normalized velocity V_ξ

Figure 2. *c, d three dimensional (3D) and countour plot (2D) of the normalized velocity* V_ξ*, on the normalized coordinates, ξ and η for the normalized time τ =0.7*

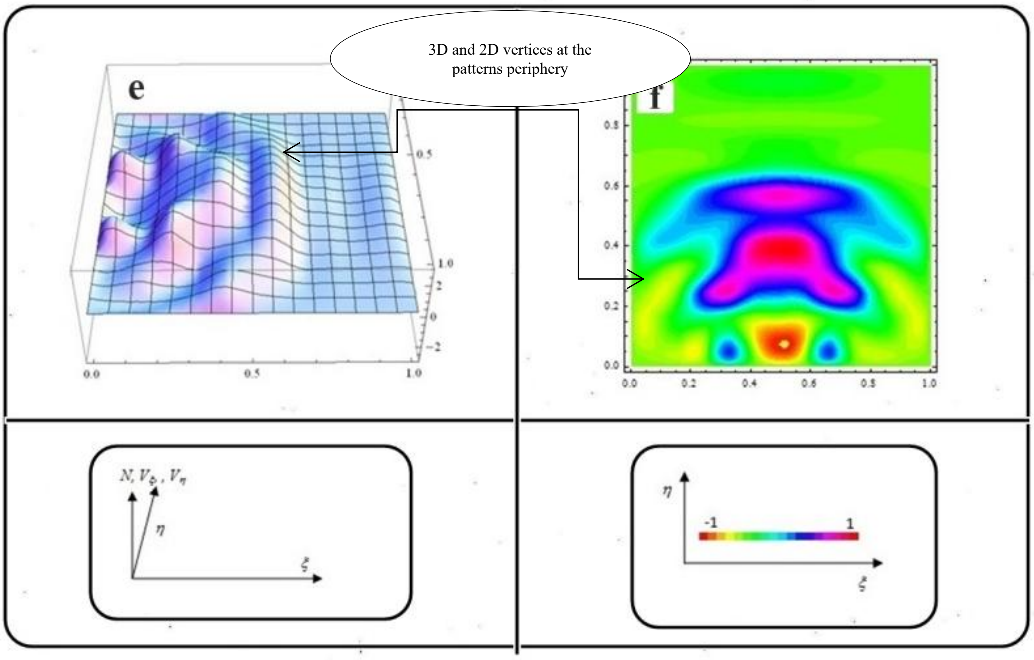


Figure 3. *e, f three dimensional (3D) and countour plot (2D) of the normalized velocity V_η,, on the normalized coordinates, ξ and η for the normalized time τ =0.7*

The following results:

i) the patterns generation;

ii) the symmetry of the normalized velocity, V_ξ, with respect to the symmetry axis of the spatio-temporal Gaussian;

iii) points at the margins of the patterns for the normalized velocity field, Vη. In this situation, it is possible to see multifractal Taylor-type effects.

For complex systems, the emergence of patterns through self-organization signifies a method of self-multiplication. We believe that all of the information above suggests that complex systems work in a way that is similar to holograms.

DISCUSSION: MULTIFRACTAL HYDRODYNAMIC MODELING AND ITS APPLICATIONS IN ENVIRONMENTAL ENGINEERING, CHEMICAL ENGINEERING, AND BIOTECHNOLOGY

The hydrodynamic multifractal model derived from Scale Relativity Theory (SRT) offers a powerful theoretical and computational framework capable of elucidating the nonlinear, self-organizing, and scale-dependent dynamics characteristic of complex systems across a wide spectrum of disciplines. The capacity of SRT to couple local fluctuations with global structures through self-similar dynamics renders it particularly relevant for understanding processes where classical continuum mechanics fails to capture emergent behavior. In this section, we explore the prospective applications of this model in three engineering and scientific domains-environmental engineering, chemical engineering, and biotechnology-emphasizing the multifractal, self-similar, and holographic nature of the systems encountered therein.

Environmental Engineering Applications

Environmental systems, particularly those involving fluid flows, pollutant dispersion, and energy transfer in atmospheric or aquatic environments, are inherently multifractal. Their behaviors are governed by nonlinear interactions across multiple scales, often exhibiting turbulence, intermittency, and self-organization. The multifractal hydrodynamic model formulated under SRT provides a theoretical foundation to analyze these processes beyond the constraints of standard Navier-Stokes dynamics [17].

Turbulent Transport and Mixing Phenomena

Pollutant transport in rivers, oceans, and the atmosphere often exhibits anomalous diffusion, deviating from classical Fickian behavior [18]. Through the multifractal velocity field decomposition (Eqs. 3 a, b), the SRT-based hydrodynamic model can describe the coexistence of regions with differing fractal dimensions, each corresponding to local scaling laws of turbulence intensity or pollutant concentration. The singularity spectrum $f(\alpha)$effectively acts as a diagnostic tool for identifying universal scaling regimes in environmental turbulence. Numerical simulations grounded in this framework could provide a more accurate description of dispersion fronts, vortex persistence, and pollutant clustering, essential for improving predictive environmental models.

Groundwater Flow and Porous Media Transport

Groundwater movement through heterogeneous geological structures presents scale-dependent permeability distributions and non-Gaussian flow paths [19]. The multifractal Madelung scenario captures such non-differentiable dynamics through its fractal potential Q, which can represent microstructural irregularities in porous media. This allows for modeling preferential pathways, percolation thresholds, and anomalous retention times, which are crucial for remediation strategies and the management of contaminant migration in aquifers. Furthermore, self-similar scaling of hydraulic conductivity, observed empirically in sedimentary formations, aligns naturally with the SRT framework.

Atmospheric and Climate Modeling

The SRT model's ability to encapsulate vortex dynamics and multifractal instabilities is also pertinent to atmospheric sciences. Cloud formation, convective turbulence, and energy cascades exhibit multifractal scaling properties that traditional meteorological models oversimplify [20]. Integrating SRT-based hydrodynamics into atmospheric modeling could refine the understanding of cloud microphysics, aerosol aggregation, and even climate feedback mechanisms by linking local perturbations to global atmospheric structures through scale covariance.

Applications in Chemical Engineering

Chemical engineering processes-ranging from reaction kinetics to multiphase transport-often unfold in domains where conventional differential models fail to represent scale-dependent fluctuations [21]. The multifractal hydrodynamic approach offers an integrative description that captures emergent order, instabilities, and transitions between laminar and turbulent states in reactive media.

Multiphase Flow and Reactive Systems

The Madelung-type equations (Eqs. 3 a, b) allow for coupling between differentiable and non-differentiable velocity components, which can model the coexistence of laminar and turbulent zones within reactors or pipelines. This is particularly relevant in processes involving gas-liquid-solid interactions [22], where reaction fronts propagate nonlinearly, and spatial self-organization emerges spontaneously. The multifractal potential Q acts analogously to a scale-dependent "reactive pressure," linking microstructural dynamics to macroscopic performance metrics such as conversion efficiency and selectivity.

Fractal Catalysis and Surface Reactions

Catalytic reactions occurring on irregular or nanostructured surfaces inherently involve fractal geometries [23]. Traditional kinetic models, which assume uniform reaction rates, fail to represent the heterogeneity of active sites and diffusion pathways. By associating the singularity spectrum $f(\alpha)$with surface energy distributions, the SRT-based framework provides a means to model variable reaction kinetics across scales. The fractal nature of catalysts can thereby be encoded into the hydrodynamic formalism, yielding more accurate predictions of reaction rates and stability under non-equilibrium conditions. This approach also offers an avenue for optimizing catalyst morphology through multifractal design parameters.

Mass and Heat Transfer in Complex Media

The self-similar properties of transport coefficients predicted by SRT correspond well with experimental observations in packed beds, foams, and emulsions [24]. Here, traditional averaging techniques obscure local heterogeneity. Through the introduction of scale-dependent transport coefficients derived from fractal geometry, the SRT model can describe heat and mass transfer across discontinuous scales, capturing the emergence of spatio-temporal oscillations and pattern formation in reactive systems. This approach could redefine reactor design by integrating multifractal dynamics into computational fluid dynamics (CFD) simulations, offering predictive control over instability regimes.

Applications in Biotechnology

Biological and biotechnological systems are archetypes of complex adaptive systems, wherein self-organization, scale-dependence, and feedback dominate their dynamics [25]. From cellular morphogenesis to metabolic regulation and tissue engineering, the multifractal hydrodynamic model provides an innovative language for representing life's intrinsic multiscale coherence.

Cellular Dynamics and Morphogenesis

At the cellular scale, processes such as cytoplasmic streaming, membrane fluctuations, and molecular diffusion are governed by stochastic and fractal geometries [26]. The Madelung formalism translates these into coherent multifractal flows where biological structures emerge as self-organizing attractors. The fractal potential Qmay be interpreted as an internal biological "information potential," linking energy dissipation and morphogenetic order. Consequently, morphogenesis can be viewed as a scale-dependent flow toward entropy minimization under fractal constraints-an interpretation consistent with observed scaling laws in embryogenesis and tissue patterning.

Bioreactor Dynamics and Bioprocess Optimization

In bioreactors, microbial populations and enzymatic networks display dynamic heterogeneity that classical kinetic models cannot fully describe [27]. The SRT framework provides a mechanism to capture scale coupling between microscopic metabolic fluctuations and macroscopic production stability. The multifractal parameters, such as fractal dimension D_F and singularity spectrum $f(\alpha)$, can serve as quantitative indicators of metabolic coherence and process efficiency. Modeling nutrient gradients and mass transfer under SRT conditions could enable the prediction of regime transitions, for example from stable growth to oscillatory or chaotic behavior.

Protein Folding and Molecular Self-Organization

At the molecular level, protein folding represents a self-organizing process within a highly complex energy landscape [28]. The multifractal formalism allows this to be understood as a transition between attractors in a scale-dependent hydrodynamic field, with the fractal potential reflecting conformational constraints. Such an approach bridges thermodynamic and kinetic models, offering an explanation for cooperative folding events and structural stability. This could provide valuable insight into protein engineering, where multifractal descriptors may guide design toward desired folding pathways or stability conditions.

Tissue Engineering and Biofluid Mechanics

Biofluid flows-such as blood circulation or interstitial fluid transport-are non-Newtonian and fractal by nature [29]. Their analysis through multifractal hydrodynamics provides an opportunity to capture the interplay between microscale rheology and macroscale perfusion. Applications in tissue engineering include optimizing scaffold design and nutrient diffusion based on scale covariance principles, as well as understanding shear-induced morphogenesis in growing tissues. This approach could yield predictive models for vascular network formation or bioreactor perfusion efficiency, integrating structure and function across scales.

Connections between machine-learning and multifractal models

Beyond conventional finite-difference integration of the Madelung-type multifractal system (Eqs. 3 a, b) and its associated multifractal potential and singularity-spectrum parameters (Eqs. 4 a-g), machine-learning (ML) offers a complementary, data-driven route for (i) extracting robust multifractal observables from simulated or experimental fields [30] and (ii) inferring model parameters from partial, noisy measurements [31]. Concretely, supervised and self-supervised models can be trained on the numerically generated density and velocity-field realizations (e.g., the pattern formation, symmetry of V_ξ, and peripheral vertices in V_η) to perform automated pattern recognition, regime classification, and early-warning detection of transitions between universality classes encoded by $f(\alpha)$.

In parallel, physics-informed learning can be used to build fast surrogate solvers or constrained estimators that explicitly penalize violations of the multifractal conservation structure, thereby preserving the SRT-consistent coupling between differentiable and non-differentiable dynamics while learning effective closures from data. In this sense, ML does not replace the SRT framework; rather, it operationalizes it-turning multifractal descriptors (fractal dimensions, singularity spectra, and potential-driven force terms) into identifiable, optimizable quantities for prediction and control, consistent with the broader perspective that integrating SRT models with modern computational tools (including ML-based pattern recognition and data-driven parameter identification [32]) can unlock improved predictive capability in complex, multiscale systems.

Cross-Disciplinary Implications

Across all these domains, the SRT-based hydrodynamic model [16, 33, 34] introduces a unifying conceptual structure for modeling complexity. Environmental, chemical, and biological systems, though physically distinct, share core attributes: nonlinearity, scale coupling, and emergent order. The holographic interpretation derived from multifractal theory implies that information encoded at local scales reflects and influences the macroscopic behavior of the entire system. Thus, the same mathematical constructs-the multifractal potential, singularity spectra, and velocity field decomposition-can be transposed from environmental turbulence to catalytic kinetics or cellular morphogenesis. Moreover, by establishing a correspondence between fractal geometry and dynamical evolution, this model can serve as a computational bridge between deterministic and stochastic representations. This has profound implications for simulation and control, suggesting that multifractal observables could serve as universal indicators of system stability, resilience, or transition thresholds. In practice, integrating SRT models with modern computational methods (e.g., finite difference solvers, machine learning-based pattern recognition, or data-driven parameter identification) could unlock predictive capabilities across various disciplines [35, 36].

Perspectives and Future Directions

The application of multifractal hydrodynamic models in engineering and biotechnology is still in its formative stage. Experimental validation-particularly through high-resolution data capturing self-similar scaling-will be essential for translating theoretical predictions into operational tools. Nonetheless, the convergence of SRT-based modeling with advanced experimental diagnostics (particle image velocimetry, spectroscopy, or imaging techniques) may soon allow for direct measurement of fractal dimensions and multifractal spectra in real systems.

Ultimately, the incorporation of the Scale Relativity framework into applied sciences could redefine how we conceptualize and design complex systems-transforming environmental remediation, chemical synthesis, and bioprocess engineering into disciplines guided not solely by empirical heuristics, but by the intrinsic geometries of nature’s scale-dependent organization.

CONCLUSIONS

This work formulated a unified Scale Relativity Theory (SRT) framework for complex-system dynamics through two complementary multifractal representations: (i) a Schrödinger-type multifractal scenario and (ii) a Madelung-type (hydrodynamic) multifractal scenario. Within this formulation, non-differentiability and resolution dependence are treated as structural features of the dynamics, enabling a consistent linkage between local multifractal fluctuations and global organization across scales.

A central result is that the Schrödinger multifractal description recovers standard quantum-like behavior in the appropriate monofractal limit, while the Madelung multifractal system makes explicit the role of the multifractal potential and the decomposition of the velocity field into differentiable and non-differentiable components. In particular, the framework clarifies how (a) a prevailing fractal dimension supports "universal" (global) dynamical patterns, (b) a set of fractal dimensions supports "zonal" (local) patterns, and (c) the singularity spectrum provides a principled route to universality-class identification even when attractor properties differ.

Numerical integration of the plane-symmetry Madelung-type multifractal system (finite-differences, Gaussian-type perturbation, normalized variables) evidences robust nonlinear self-organization. The computed fields exhibit: (i) spontaneous pattern generation in the normalized density, (ii) symmetry properties of the normalized velocity component $\boldsymbol{V_\xi}$relative to the perturbation symmetry axis, and (iii) peripheral vertices in the $\boldsymbol{V_\eta}$field consistent with vortex-like/Taylor-type multifractal effects. These features support the interpretation that multifractal coupling can act as an intrinsic mechanism for structure formation and symmetry selection in complex flows.

Collectively, the theoretical development and simulations indicate that complex systems governed by SRT-consistent multifractal dynamics can display self-similar (holographic-type) behavior, in the sense that coherent macroscopic organization emerges from, and remains coupled to, the scale-dependent microstructure encoded in the multifractal degrees of freedom. This motivates the use of the proposed framework as a transferable modeling "template" for multiscale phenomena where classical smooth-manifold assumptions obscure emergent order.

From an applications standpoint, the multifractal hydrodynamic model provides a plausible bridge between first-principles, scale-covariant dynamics and engineering practice in environmental transport, chemical reactive/multiphase systems, and biotechnology, while also aligning naturally with modern computational strategies (e.g., physics-informed learning, surrogate modeling, and data-driven inference of multifractal observables).

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