POWER VARIANT FOR EVALUATION OF SOLICITATION OF CIRCULAR AND PLANE PLATES WITH RADIAL RIBS (II)

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Abstract. This paper continues the study of deformation and stresses state in circular and plane plates with radial ribs which are situated on one face and it tackles the power method illustrated by [1]. This paper presents the evaluation expressions of stresses both in the base plate and ribs, under the influence of an uniform distributed pressure on the face without ribs of the plate. The mentioned analysis is produced for the plate with the peripheral outline which is rigid fixed, respectively leaned simple.

Key word: ribbed plate, fixed plate, leaned simple plate

1. EVALUATION OF THE STRESSES

In the first part of the paper [2] the specific expressions for the calculus of auxiliary dimensions e (the relations (31) and (42)) and A (the relations (32) and (44)) are presented, for calculus of the displacement of the plate with radial ribs which are situated on one of its faces, under the action of an uniform pressure on the other surface, which are written under the form:

$$w_n = \frac{p \cdot A}{64 \cdot \Re} \cdot \left(r_{cr}^2 - r^2 \right)^2 \text{ - for the outline fixed plate, respectively:} \tag{45}$$

$$w_n = \frac{p \cdot A}{64 \cdot \Re} \cdot \left(\frac{5 + \nu_p}{1 + \nu_p} \cdot r_{cr}^4 - \frac{3 + \nu_p}{1 + \nu_p} \cdot r^2 + r^4 \right) \text{ - for the leaned simple plate.} \tag{46}$$

The paper [1] proposes for the calculus of σ_r the radial stresses and σ_θ the circumferential stresses, in the plane state of solicitation for smooth plate and axial for ribs, the following formula:

► for stresses of the base plate:

$$\sigma_r = -\frac{E_p \cdot (z - e)}{1 - \nu_p^2} \cdot \left(\frac{d^2 w_n}{d r^2} + \frac{\nu_p}{r} \cdot \frac{d w_n}{d r} \right); \tag{47}$$

$$\sigma_\theta = -\frac{E_p \cdot (z - e)}{1 - \nu_p^2} \cdot \left(\frac{1}{r} \cdot \frac{d w_n}{d r} + \nu_p \cdot \frac{d^2 w_n}{d r^2} \right); \tag{48}$$

► for ribs:

$$\sigma_{rn} = -E_n \cdot \frac{d^2 w}{dr^2} \cdot (z - e). \tag{49}$$

The calculus relations for the evaluation of the stresses state from the plane and circular plate and from ribs take into discussions the stresses expressions created in the base plate (52) and (53), respectively in ribs (54), as well as the equalities (50) and (51).

2. OUTLINE FIXED PLATE

► ***For the plane plate:***

- *The radial stress:*

$$\sigma_r = \frac{p \cdot E_p \cdot A}{16 \cdot \left(1 - \nu_p^2\right) \cdot \Re} \cdot (z - e) \cdot \left[\left(1 + \nu_p\right) \cdot r_{cr}^2 - \left(3 + \nu_p\right) \cdot r^2\right], \tag{50}$$

with corresponding particularities:

$$r = 0; \qquad \sigma_r = \frac{p \cdot E_p \cdot A \cdot (z - e)}{16 \cdot \Re \cdot \left(1 - \nu_p\right)} \cdot r_{cr}^2, \tag{51}$$

therefore at $\nu_p = 0{,}3$ (oțel), $\sigma_r = 0{,}089 \cdot \frac{p \cdot E_p \cdot A \cdot (z - e)}{\Re} \cdot r_{cr}^2$, while at:

$$r = r_{cr}; \qquad \sigma_r = -\frac{p \cdot E_p \cdot A \cdot (z - e)}{8 \cdot \Re \cdot \left(1 - \nu_p\right)} \cdot r_{cr}^2, \tag{52}$$

respectively, for $\nu_p = 0{,}3$, $\sigma_r = -0{,}137 \cdot \frac{p \cdot E_p \cdot A \cdot (z - e)}{\Re} \cdot r_{cr}^2$.

Note: The change of radial stress sign logway current radius of the plate is produced when:

$$\left(1 + \nu_p\right) \cdot r_{cr}^2 - \left(3 + \nu_p\right) \cdot r^2 = 0, \tag{53}$$

this carries at:

$$r = r_{cr} \cdot \sqrt{\frac{1 + \nu_p}{3 + \nu_p}}, \tag{54}$$

respectively, for $\nu_p = 0{,}3$, $r = 0{,}628 \cdot r_{cr}$.

- *The circumferential stress:*

$$\sigma_\theta = \frac{p \cdot E_p \cdot A}{16 \cdot \left(1-\nu_p^2\right) \cdot \Re} \cdot (z-e) \cdot \left[\left(1+\nu_p\right) \cdot r_{cr}^2 - \left(1+3\cdot\nu_p\right)\cdot r^2\right], \tag{55}$$

expression which take the same form (56) for $r = 0$, respectively the particularized expression for $\nu_p = 0,3$, while at:

$$r = r_{cr}\,; \qquad \sigma_\theta = -\frac{p \cdot E_p \cdot A \cdot (z-e) \cdot \nu_p}{8 \cdot \Re \cdot \left(1-\nu_p\right)} \cdot r_{cr}^2\,, \tag{56}$$

which for $\nu_p = 0,3$ it become $\sigma_r = -0,041 \cdot \dfrac{p \cdot E_p \cdot A \cdot (z-e)}{\Re} \cdot r_{cr}^2$.

<u>Note:</u> The change of the circumferential stress sign longway current radius of the plate, when:

$$\left(1+\nu_p\right) \cdot r_{cr}^2 - \left(1+3\cdot\nu_p\right) \cdot r^2 = 0\,, \tag{57}$$

this carries at:

$$r = r_{cr} \cdot \sqrt{\frac{1+\nu_p}{1+3\cdot\nu_p}}\,, \tag{58}$$

respectively, for $\nu_p = 0,3$, $r = 0,684 \cdot r_{cr}$.

From the analyses of up expressions we remark the fact that, in absolute value, the radial stress is bigger than that circumferential and the both have the same intensity in the center of the plate.
In accordance with nature of the plate material and the option of the designer of such constructive element, the equivalent stresses can be calculated after:

- *<u>the third resistance theory:</u>*

$$\sigma_{ech}^{III} = \frac{p \cdot E_p \cdot A \cdot (z-e)}{8 \cdot \Re \cdot \left(1-\nu_p\right)} \cdot r_{cr}^2\,, \tag{59}$$

respectively, for $\nu_p = 0,3$, $\sigma_{ech}^{III} = 0,137 \cdot \dfrac{p \cdot E_p \cdot A \cdot (z-e)}{\Re} \cdot r_{cr}^2$.

- *<u>the fourth resistance theory</u>*:
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$$\sigma_{ech}^{IV} = \frac{p \cdot E_p \cdot A \cdot (z-e)}{16 \cdot \Re \cdot \left(1-\nu_p^2\right)} \cdot \sqrt{\left(1+\nu_p\right)^2 \cdot \left(r_{cr}^2 - 4 \cdot r^2\right) \cdot r_{cr}^2 + \left(13 \cdot \nu_p^2 + 22 \cdot \nu_p + 13\right) \cdot r^4}\,, \tag{60}$$

becaming for:

$$r = 0\,; \qquad \sigma_{ech}^{IV} = \frac{p \cdot E_p \cdot A \cdot (z-e)}{16 \cdot \Re \cdot \left(1-\nu_p\right)} \cdot r_{cr}^2\,; \tag{61}$$

$$r = r_{cr}\ ;\ \sigma_{ech}^{IV} = 0{,}088 \cdot \frac{\sqrt{5 \cdot \nu_p^2 + 8 \cdot \nu_p + 5}}{1 - \nu_p^2} \cdot \frac{p \cdot E_p \cdot A \cdot (z - e)}{\Re} \cdot r_{cr}^2 . \tag{62}$$

For a steel plate, with $\nu_p = 0{,}3$, the equalities (61) and (62) became:

$$r = 0\ ; \quad \sigma_{ech}^{IV} = 0{,}089 \cdot \frac{p \cdot E_p \cdot A \cdot (z - e)}{\Re} \cdot r_{cr}^2\ ; \tag{63}$$

$$r = r_{cr}\ ;\ \sigma_{ech}^{IV} = 0{,}272 \cdot \frac{p \cdot E_p \cdot A \cdot (z - e)}{\Re} \cdot r_{cr}^2 . \tag{64}$$

Note: a) We find out that the equivalent stresses calculated with the both resistance theories have the same value in the center of the plate, while the inferred value with the fourth resistance theory is about double against that evaluated with the third resistance theory.

b) In the previous expressions we'll take into consideration, at equivalent stresses calculus, the absolute value of the parenthesis $(z - e)$, namely, on the surfaces of the base plate $\left| \left(\pm \frac{\delta_p}{2} - e \right) \right|$, where the dimension e is given of formula (31).

c) The dimension A is calculated with the expression (32).

► ***For ribs:***

The formula of the radial stress from ribs result introducing the expression of the second fluxion of the plate displacement, written under the form:

$$\sigma_{rn} = \frac{p \cdot E_n \cdot A \cdot (z - e)}{16 \cdot \Re} \cdot \left(r_{cr}^2 - 3 \cdot r^2 \right), \tag{65}$$

with the particularities for:

$$r = 0\ ; \quad \sigma_{rn} = \frac{p \cdot E_n \cdot A \cdot (z - e)}{16 \cdot \Re} \cdot r_{cr}^2\ ; \tag{66}$$

$$r = r_{cr}\ ;\ \sigma_{rn} = -\frac{p \cdot E_n \cdot A \cdot (z - e)}{8 \cdot \Re} \cdot r_{cr}^2 . \tag{67}$$

We find out in the radius long way, the radial stress from ribs change its meaning and the value of the respective radius is given of the equality $r = 0{,}577 \cdot r_{cr}$.

3. OUTLINE LEANED SIMPLE PLATE

If we adopt the same calculus procedure we adopt the following formula for stresses.

► ***For plane plate:***

- *The radial stress:*

$$\sigma_r = -\frac{p \cdot E_p \cdot A}{16 \cdot \left(1-\nu_p^2\right) \cdot \Re} \cdot (z-e) \cdot \left[\frac{\nu_p^2 + 2 \cdot \nu_p + 3}{1+\nu_p} \cdot r_{cr}^2 - \left(3+\nu_p\right) \cdot r^2\right], \tag{68}$$

with corresponding particularities:

$$r = 0; \qquad \sigma_r = -\frac{\nu_p^2 + 2 \cdot \nu_p + 3}{16 \cdot \left(1+\nu_p\right)^2 \cdot \left(1-\nu_p\right)} \cdot \frac{p \cdot E_p \cdot A \cdot (z-e)}{16 \cdot \Re \cdot \left(1-\nu_p\right)} \cdot r_{cr}^2, \tag{69}$$

such for $\nu_p = 0{,}3$ (steel), $\sigma_r = -0{,}195 \cdot \dfrac{p \cdot E_p \cdot A \cdot (z-e)}{\Re} \cdot r_{cr}^2$,while at:

$$r = r_{cr}; \qquad \sigma_r = \frac{\nu_p}{8 \cdot \left(1+\nu_p\right)^2 \cdot \left(1-\nu_p\right)} \cdot \frac{p \cdot E_p \cdot A \cdot (z-e)}{8 \cdot \Re \cdot \left(1-\nu_p\right)} \cdot r_{cr}^2, \tag{70}$$

respectively, for $\nu_p = 0{,}3$, $\sigma_r = 0{,}032 \cdot \dfrac{p \cdot E_p \cdot A \cdot (z-e)}{\Re} \cdot r_{cr}^2$.

<u>**Note**</u>: Radial stress change the sign long way the current radius of the plate when:

$$\frac{\nu_p^2 + 2 \cdot \nu_p + 3}{1+\nu_p} \cdot r_{cr}^2 - \left(3+\nu_p\right) \cdot r^2 = 0, \tag{71}$$

this permits the establishment of the equality:

$$r = r_{cr} \cdot \sqrt{\frac{\nu_p^2 + 2 \cdot \nu_p + 3}{\left(1+\nu_p\right) \cdot \left(3+\nu_p\right)}}, \tag{72}$$

respectively, for $\nu_p = 0{,}3$, $r = 0{,}927 \cdot r_{cr}$.

- *<u>The circumferential stress:</u>*

$$\sigma_\theta = -\frac{p \cdot E_p \cdot A}{16 \cdot \left(1-\nu_p^2\right) \cdot \Re} \cdot (z-e) \cdot \left[\left(3+\nu_p\right) \cdot r_{cr}^2 - \frac{3 \cdot \nu_p^2 + 4 \cdot \nu_p + 3}{1+\nu_p} \cdot r^2\right], \tag{73}$$

With partcular forms for:

$$r = 0; \quad \sigma_\theta = -\frac{3+\nu_p}{16 \cdot \left(1-\nu_p^2\right)} \cdot \frac{p \cdot E_p \cdot A}{\Re} \cdot (z-e) \cdot r_{cr}^2, \tag{74}$$

$$r = r_{cr}\,; \quad \sigma_\theta = \frac{\nu_p^2}{8\cdot(1+\nu_p)^2\cdot(1-\nu_p)}\cdot\frac{p\cdot E_p\cdot A\cdot(z-e)}{\Re}\cdot r_{cr}^2\,, \tag{75}$$

what, for $\nu_p = 0{,}3$ became:

$$\sigma_\theta = -\,0{,}227\cdot\frac{p\cdot E_p\cdot A\cdot(z-e)}{\Re}\cdot r_{cr}^2\,,\ \text{for } r = 0\,,\ \text{respectively:}$$

$$\sigma_\theta = \ 0{,}01\cdot\frac{p\cdot E_p\cdot A\cdot(z-e)}{\Re}\cdot r_{cr}^2\,,\ \text{for } r = r_{cr}\,.$$

Note: From the (68) equality analyzes we find out that the change of circumferential stress sign long way the current radius of the plate is produced when:

$$(3+\nu_p)\cdot r_{cr}^2 - \frac{3\cdot\nu_p^2 + 4\cdot\nu_p + 3}{1+\nu_p}\cdot r^2 = 0\,, \tag{76}$$

this carries at:

$$r = r_{cr}\cdot\sqrt{\frac{(3+\nu_p)\cdot(1+\nu_p)}{3\cdot\nu_p^2 + 4\cdot\nu_p + 3}}\,, \tag{77}$$

respectively, for $\nu_p = 0{,}3$, $r = 0{,}98\cdot r_{cr}$.

The equivalent stresses are calculated like in the previous case, helping with:

- *the third resistance theory*:

$$\sigma_{ech}^{III} = \frac{p\cdot E_p\cdot A}{16\cdot(1-\nu_p^2)\cdot\Re}\cdot|z-e|\cdot\left[\frac{\nu_p^2 + 2\cdot\nu_p + 3}{1+\nu_p}\cdot r_{cr}^2 - (3+\nu_p)\cdot r^2\right], \tag{78}$$

for the current radius of the plate, taking over the corresponding expression of the radial stress, which has a bigger value than the circumferential stress, for anything:

$$r \,\langle\, \sqrt{\frac{1}{\nu_p}}\cdot r_{cr}\,, \tag{79}$$

respectively, for $\nu_p = 0{,}3$, $r \,\langle\, 1{,}826\cdot r_{cr}$.

The maximal value of the stress, in this case, has the form:

$$\sigma_{ech,M}^{III} = \frac{\nu_p^2 + 2\cdot\nu_p + 3}{16\cdot(1+\nu_p)^2\cdot(1-\nu_p)}\cdot\frac{p\cdot E_p\cdot A\cdot|z-e|}{\Re}\cdot r_{cr}^2\,, \tag{80}$$

which for $\nu_p = 0{,}3$ became:

$$\sigma_{ech,M}^{III} = 0{,}195 \cdot \frac{p \cdot E_p \cdot A \cdot |z-e|}{\Re} \cdot r_{cr}^2 . \tag{81}$$

- *the fourth resistance theory:*

$$\sigma_{ech}^{IV} = \frac{p \cdot E_p \cdot A \cdot |z-e|}{\left(1+\nu_p\right) \cdot \Re} \cdot \sqrt{B} , \tag{82}$$

where:

$$B = \begin{bmatrix} \left(\nu_p^2 + 4 \cdot \nu_p + 3\right) \cdot r_{cr}^2 - \\ -\left(3 \cdot \nu_p^2 + 4 \cdot \nu_p + 3\right) \cdot r^2 \end{bmatrix}^2 - 2 \cdot \nu_p \cdot \begin{bmatrix} \left(\nu_p^2 + 2 \cdot \nu_p + 3\right) \cdot r_{cr}^2 - \\ -\left(\nu_p^2 + 4 \cdot \nu_p + 3\right) \cdot r^2 \end{bmatrix} \cdot \left(r_{cr}^2 - \nu_p \cdot r^2\right). \tag{83}$$

From analyzes of the (82) and (83) relations, we easily find out that the maximal value of the equivalent stress, which is calculated helping the fourth theory, is in the center of the plate, for $r = 0$, when:

$$\sigma_{ech}^{IV} = \frac{p \cdot E_p \cdot A \cdot |z-e|}{\left(1+\nu_p\right) \cdot \Re} \cdot r_{cr}^2 \cdot \sqrt{\left(\nu_p^2 + 4 \cdot \nu_p + 3\right)^2 - 2 \cdot \nu_p \cdot \left(\nu_p^2 + 2 \cdot \nu_p + 3\right)} , \tag{84}$$

or, for $\nu_p = 0{,}3$, $\sigma_{ech}^{IV} = 3{,}095 \cdot \dfrac{p \cdot E_p \cdot A \cdot |z-e|}{\Re} \cdot r_{cr}^2$.

Note: Taking into account the ribs can't come across into the plate center, technological justifications, the circumference of r_a radius, who they can be ribbed by a central tube, for example, is characterized by formula:

$$r_a \geq \frac{n}{2 \cdot \pi} \cdot \left[5 \cdot max\left\{\delta_p ; \delta_n\right\} + \delta_n\right]. \tag{85}$$

► ***For ribs:***

The (49) equality, after we use the second fluxion of the displacement of the smooth plate, correlated with that of the ribbed plate, carries at:

$$\sigma_{rn} = -\frac{p \cdot E_n \cdot A \cdot (z-e)}{16 \cdot \Re} \cdot \left(\frac{3+\nu_p}{1+\nu_p} \cdot r_{cr}^2 - 3 \cdot r^2\right), \tag{86}$$

which show the maximal value of the stress (in absolute value) is fond at $r = 0$, where:

$$\sigma_{rn} = \frac{p \cdot E_n \cdot A \cdot |z-e|}{16 \cdot \Re} \cdot \frac{3+\nu_p}{1+\nu_p} \cdot r_{cr}^2 . \tag{87}$$

4. CONCLUSIONS

Further the study about the deformation and stresses states which are created in the circular and plane plates having radial and equivalent ribs on one of the surfaces, under the action of a uniform distributed pressure on the opposite surface, in this paper are presented the expressions of the developed stresses in the base plate and in the ribs, accepting two clamping modes of the plates outline: rigid fastening and simple leaning. For the evaluation of the equivalent stresses the possible utilization of the third or the fourth resistance theories is specified. A constant geometry of the ribs, long way these, was presented in calculus, in this case.

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