THEORETICAL ASPECTS CONCERNING PREDICTION OF GRAVITY FLOW TIME THROUGH ORIFICES OF THE GRANULAR SOLIDS MATERIALS

CĂSĂNDROIU TUDOR, VOICU GHEORGHE, IONUŢ ARSENE

"Politehnica" University of Bucharest, Faculty of Biotechnical Systems Engineering

Abstract. The mathematical models for the gravity flow times of the noncohesive granular materials (with seeds application) are theoretically grounded and developed in this paper. There are proposed concrete models which emphasize the main parameters influencing the flow, as: the bin geometry and the physical characteristics of the granular material. There is done a numerical application which allows us the follow the needed calculus steps, suggesting us the procedure for the elaboration of an calculus software.

Keywords: granular materials, grain, gravity flow, flow rate, flow time, theoretical models

1. INTRODUCTION

Inside handling operations of granular noncohesive materials, particularly cereals, gravitational flow frequently appears during their transfer, when particles are under the influence of their own weight and are discharged, [1,2,4,5]. The transfer of granular materials from their storage bins is done by a flow through orifices of different sizes and geometries.

Aspects concerning the flow of granular materials through orifices from storage bins are present in dosing-packaging-filling systems, in bins unloading, in supplying materials from bins in complex technological processes, during design activity, for online controlling of installations, in the activity of automatic control assisted by computer, etc. [4].

Beginning with year 1900, many researchers have studied the phenomenon of granular materials flow through orifices, especially to find (develop) some mathematical models that would allow the evaluation, through calculation, of output flows as near as possible to experimental data [1,2,4,5].

Output flows depend on orifice and bin geometry, on physical properties of the granular material (internal friction, dimensions and geometrical shape of particles, bulk density, humidity content), on the amount of time they are stored and on environment conditions [4,5].

One property of granular materials flow that is useful in engineering calculations, besides output flow, is the flow time of a given quantity from a material, in known conditions [3].

In order to test granular materials for the two above-mentioned characteristics, some companies have realized suitable devices that can be purchased [6].

The objectives of this article are the theoretical elaboration of a mathematical model to calculate the gravitational flow-time of seeds through orifices on the horizontal plane and applications on known experimental cases from articles published in specific reviews.

2. THEORETICAL CONSIDERATIONS

2.1. Approximate Prediction

In an approximate meaning, in order to evaluate the gravitational flow-time t(s) of a given quantity m(kg) from a granular material through an orifice, it is supposed that the mass flow rate Q(kg/s) of the material flow through orifices is constant and known, and thus:

$$t = m / Q \qquad (1)$$

This assumption is based on research results in which many researchers unanimously accept that the mass flow rate Q does not depend on the granular material height from inside the bin and on the bin diameter D, if condition [5] is fulfilled: $D > 1{,}5D_0$, or more general if $D - D_0 > 30d_p$, in which D_0 is the orifice diameter, and d_p is the equivalent diameter for the particles of the granular material ($d_p = \left(\frac{6 \cdot m}{\pi \cdot \rho}\right)^{1/3}$) in which m_p is the mean value for the particle's mass, and ρ is the average density of the material.

Between mathematical models frequently used for an acceptable evaluation, through calculation, of flow mass gravitational outputs in seeds case, we mention [1,2,4,5]:

- the model developed in year 1959 by Fowler-Glastonbury, [4,5]:

$$Q = C_1 \rho_v A \sqrt{2 g D_o} \left(\frac{D_o}{d_p}\right)^{0.185} \quad \text{(kg/s)} \qquad (2)$$

- the model developed in 1961 by Beverloo, [4,5]:

$$Q = C_2 \rho_v \sqrt{g} \left(D_o - 1.4 d_p\right)^{2.5} \quad \text{(kg/s)} \qquad (3)$$

- the model developed in 1990-1991 by Chang and Converse, [1,2]:

$$Q = C_3 \rho_v A D_h^n \quad \text{(kg/s)} \qquad (4)$$

- the model developed in 1995 by Wang with practical applications especially for soybean flour, [5]:

$$Q = C_4 \rho_v \sqrt{g}\, D_o^{2.5} \left(\frac{D}{D_o}\right)^{\alpha} \left(\frac{\theta}{\psi_o}\right)^{\beta} \quad \text{(kg/s)} \qquad (5)$$

In equations (2), (3), (4) and (5) the meaning of terms is: C_1, C_2, C_3, C_4 coefficients (C_1=0,236, C_2=0,583, C_3 and C_4 are detailed in [1,2], respectively [5] depending on seeds type); ρ_v – the bulk density of the seeds (kg/m^3); A – the area of the orifice (m^2); g – gravitational acceleration (m/s^2); D_h – the hydraulic diameter of the orifice (m); θ - the angle made by inclined orifice wall with the horizontal; ψ_0 – the angle of repose for the granular material; α and β - coefficients which depend on humidity content, [5].

In practical calculation applications, the evaluation of the mass flow rate using equation (3) is the closest with data obtained in measurements, for a large scale of seeds, [4].

Taking into account this remark, it is to be found that the flow-time of a given amount of seeds can be estimated with an acceptable accuracy, using the equation:

$$t = \frac{m}{C_2 \rho_v \sqrt{g} \left(D_o - 1.4 d_p\right)^{2.5}} \quad \text{(s)} \qquad (6)$$

In the special case in which the material-mass is related to the full bin, then from equation (6) will result the empty-time t_g for a bin through gravitational flow:

$$t_g = \frac{0.548\,V}{\left(D_o - 1.4\,d_p\right)^{2.5}} \quad \text{(s)} \tag{7}$$

in which V(m^3) represent the volume of the bin.
If we define the specific empty-time $t_{sg} = t_g/V$ (s/m^3), from equation (7) will result:

$$t_{sg} = \frac{t_g}{V} = \frac{0.548}{\left(D_o - 1.4\,d_p\right)^{2.5}} \quad \text{(s/m}^3\text{)} \tag{8}$$

On the basis of estimation with equation (8) of the specific flow time through orifices t_{sg}, it can be estimated the flow-time (t_1) of a granular material from any volume fraction V_1 of the bin:

$$t_1 = V_1\, t_{sg} \tag{9}$$

For this approximate evaluation of the specific flow time t_{sg} , according to equation (8), in figure 1 it is graphically plotted the variation of t_{sg} versus diameter of the orifice D_0 and for more values of the particle diameter d_p , from the domain of the frequently used values, [4].
It can be easily determined that t_{sg} decreases with the increase of diameter D_0 and with the decrease of particle diameter d_p.

The same conclusion can be also drawn from figure 2, in which it has been graphically plotted the variation of t_{sg} versus particle diameter d_p for more values of the particle diameter D_0.

As presented in [3], the conclusion is valid for d_p > 0.15 mm, because at lower values the angle of internal friction between particles increases and the susceptibility that the particles adhere to each other and to the walls is higher, which determines that at values of d_p < 0.15 mm the specific flow time increases.

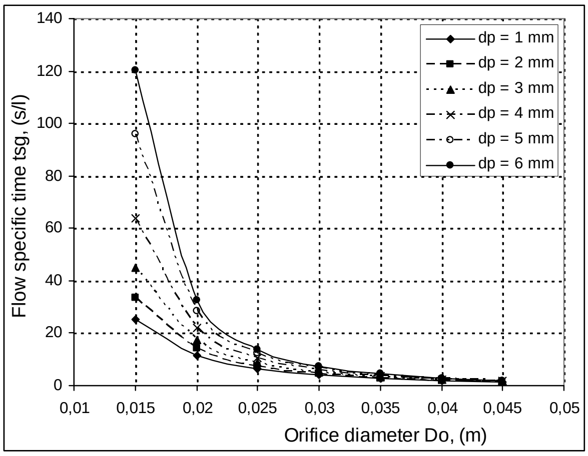

Fig.1. Variation of the flow specific time t_{sg} vs. orifice diameter D_o for various values of the particle diameter d_p

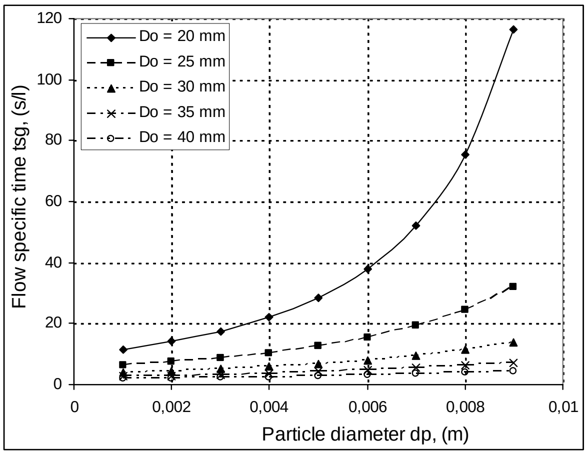

Fig.2. Variation of the flow specific time t_{sg} vs. particle diameter d_p for various values of the orifice diameter D_o

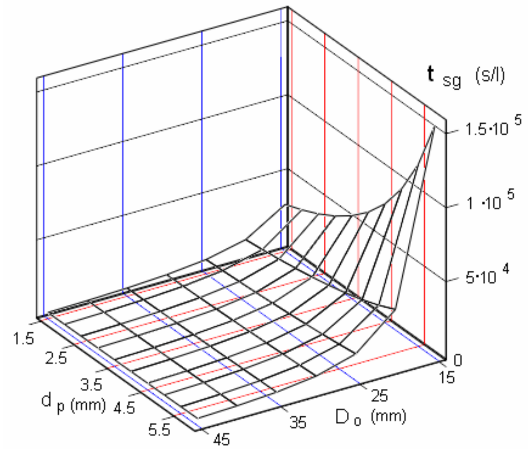

Fig.3. Variation of the flow specific time t_{sg} vs. orifice diameter D_o and particle diameter d_p

In figure 3 it has been plotted the 3-D simultaneously graphic variation of the specific flow time t_{sg} vs. orifice diameter D_o and particle diameter d_p.

It can be easily stated the same conclusion mentioned above. Using this graphic it can be approximately predicted the value of t_{sg} (s/l) for given values of D_o (mm) and d_p (mm).

Those graphics are useful because the values of t_{sg} (s/l) can be easily determined and then, using equation (9), the flow times corresponding to requested volumes may be determined (l) and by these values the capacity to flow of the granular material is characterized.

2.2. Analytical Prediction

For an accurate flow-time evaluation, it is necessary to take into account the main parameters that influence the flow, like: bin's geometry, the shape and sizes of the orifice, the physical properties of the granular material, [1-5].

In this case we will use, in order to evaluate the mass flow rate for a gravitational flow through orifices for granular materials, the general equation, used and developed in many articles about granular noncohesive materials from the field of chemistry and plastics [3]. We will take this equation as valid for the case of granular materials in the agricultural field:

$$Q = K\, p^m\, D_o^n \quad \text{(kg/s)} \tag{10}$$

in which K, m, n are material constants (parameters); p – the pressure generated by the granular level under the gravitational effect, at the orifice level through which flow is taking place (Pa); D_0(m) – the diameter of the orifice. By processing some experimental data [3], it resulted m ≈ 0,5 and n ≈ 2,5. We will use these values, and will obtain:

$$Q = K\, p^{0.5}\, D_o^{2.5} \quad \text{(kg/s)} \tag{11}$$

For the pressure p generated by the granular level under the gravitational effect, we will use the Janssen equation, [4]:

$$p = \frac{\rho_v\, g\, R_h}{\mu\, k} \cdot \left(1 - e^{-\frac{\mu k}{R_h} \cdot H} \right) \quad \text{(Pa)} \tag{12}$$

in which R_h(m) is the hydraulic radius of the bin's cross-section; μ - the friction coefficient of the granular material with bin's walls; H(m) – the height of the granular material level versus orifice level; k – the mobility coefficient of the granular material, depending on its angle of repose ψ_0, defined by equation [4]:

$$k = \frac{1 - \sin \psi_o}{1 + \sin \psi_o} \tag{13}$$

By replacing the expression from equation (12) in equation (11), we will obtain the equation for the mass volume estimation for a granular material:

$$Q = K\left(\frac{\rho_v\, g\, R_h}{\mu\, k}\right)^{0.5} \cdot \left(1 - e^{-\frac{\mu \cdot k}{R_h}\cdot H}\right)^{0.5} \cdot D_o^{2.5} \ \text{(kg/s)} \tag{14}$$

In order to estimate the flow-rate for a given granular material, we will take into account the infinitesimally elementary material quantity dm contained in a material-level from bin's cross-section, placed at level x related to the orifice, of a certain dx thickness:

$$dm = \rho_v\, A(x) \cdot dx \tag{15}$$

in which A(x) is the area of the cross-section through bin at the x level related to the orifice.

If, for this regarded position of the section, the mass flow output through the orifice (Q) is given by equation (14) in which H=x and $R_h = R_x = A(x)/L_x$ (L_x – the perimeter for bin' section at level x), then:

$$\rho_v\, A(x) \cdot dx = Q \cdot dt \tag{16}$$

The equation (16) is the differential equation with separable variables, by integration of which it is obtained the flow-time of a quantity of material contained between levels H_1 and H, thus:

$$t = \int_{H_1}^{H} \frac{\rho_v\, A(x)}{Q} \cdot dx \tag{17}$$

By replacing the expression from equation (14) in equation (17), after all the calculations are made, it is obtained, for the general case of bin's geometry:

$$t = \frac{\rho_v^{0.5}}{K}\left(\frac{g}{\mu\, k}\right)^{-0.5} D_o^{-2.5} \int_{H_1}^{H} A(x)\, R_x^{-0.5}\left(1 - e^{-\frac{\mu \cdot k}{R_x}\cdot x}\right)^{-0.5} \cdot dx \tag{18}$$

When the bin' surface is generated by a revolution surface through rotation of a curve of which profile is defined by the radius with the rotation axis r_x at the x level from the orifice, when $A(x) = \pi \cdot r_x^2$, it is obtained:

$$t = \frac{\sqrt{2}\,\pi\,\rho_v^{0.5}}{K}\left(\frac{g}{\mu\, k}\right)^{-0.5} D_o^{-2.5} \int_{H_1}^{H} r_x^{1.5}\left(1 - e^{-\frac{2\,\mu \cdot k}{r + x \cdot \tan\alpha}\cdot x}\right)^{-0.5} \cdot dx \tag{19}$$

For the particular case of a bin trunk-cone shaped with basis radiuses r and R and the small basis on the lower side that represents the output orifice ($D_o = 2 \cdot r$), taking into account that: $r_x = r + x \cdot \tan\alpha$ (2α is the angle at the top of the cone for the trunk-cone surface), after replacing in equation (19) it is obtained:

$$t = \frac{\pi \rho_v^{0.5}}{4K}\left(\frac{g}{\mu k}\right)^{-0.5} r^{-2.5}\int_{H_1}^{H}\left(r + x\cdot\tan\alpha\right)^{1.5}\left(1 - e^{-\frac{2\mu\cdot k}{r+x\cdot\tan\alpha}\cdot x}\right)^{-0.5}\cdot dx \qquad (20)$$

When the empty-time t_g is needed, in equations (18), (19), (20) the lower limit of the integral is replaced, $H_1 = 0$. For the certain case of a bin with known dimensions and geometrical shape and a given granular material through its parameters ρ_v, μ and k, in equations (18), (19) and (20) the expressions of all the functions that depend on x are established and a numerical calculation is done for the integrals, using an adequate calculation software (like MathCad), and also a numerical calculation of the flow-time t.

For the particular case of a cylindrical circular bin of a diameter D, in equation (19), the integral can be solved, taking into account that $r_x = D/2$ and, after the replacement, it would be obtained:

$$t = \frac{\pi \rho_v^{0.5} D^{1.5}}{2K}\left(\frac{\mu k}{g}\right)^{0.5} D_o^{-2.5}\int_{H_1}^{H}\left(1 - e^{-\frac{4\mu\cdot k}{D}\cdot x}\right)^{-0.5} dx \text{ (s)} \qquad (21)$$

After the integral is solved, taking into account that the undefined integral $\int\left(1-e^{-a x}\right)^{0.5} dx = -\frac{2}{a}\ln\left[\tan\left(\frac{1}{2}\arcsin\sqrt{e^{-a x}}\right)\right]$, it would be obtained:

$$t = \frac{\pi \rho_v^{0.5}}{4K}\frac{1}{\left(\mu k g\right)^{0.5}}\left(\frac{D}{D_o}\right)^{2.5}\ln\left[\frac{\tan\left(\frac{1}{2}\arcsin\sqrt{e^{-\frac{4\mu k}{D}\cdot H_1}}\right)}{\tan\left(\frac{1}{2}\arcsin\sqrt{e^{-\frac{4\mu k}{D}\cdot H}}\right)}\right] \text{ (s)} \qquad (22)$$

In order to calculate the empty-time of the cylindrical bin, $H_1 = 0$ is replaced in equation (22) and it would be obtained:

$$t = \frac{\pi \rho_v^{0.5}}{4K}\frac{1}{\left(\mu k g\right)^{0.5}}\left(\frac{D}{D_o}\right)^{2.5}\ln\left[\tan\left(\frac{1}{2}\arcsin\sqrt{e^{-\frac{4\mu k}{D}\cdot H}}\right)\right]^{-1} \text{ (s)} \qquad (23)$$

The calculation equations for the flow-time (19), (20), (22) and (23) are presenting the parameters linked with bin's geometry and dimensions, with those of the orifice through D, D_o, H, α, and of the granular material physical characteristics μ, ψ_o, k, ρ_v, K by which this time is influenced.

In order to make all the calculations, it is necessary to know the coefficient K that depends on granular material and that is obtained from experimental measurements. We will try to evaluate this coefficient through calculations for wheat seeds, and it follows to be determined through experiments.

2.3. A calculation application

We will use, for the calculation, wheat seeds with the physical properties considered in [4], with the cylindrical sizes of the bin used in experiments in [5] and those of a trunk-cone bin used at the testing device for the granular materials flow [3,6], namely: ρ_v =800 kg/m^3; μ = 0.45; ψ_o = 27^o; D_o = 25.3 mm; D = 300 mm; H = 900 mm; α = 20^o; r = D_o/2 = 12.65 mm; R = 55.5 mm; d_p = 4.1 mm.

For the calculation of the mass flow rate through the orifice with the diameter D_0, we will use the equation (3), which gives the most appropriate values to the real measurements data. It is obtained Q = 78 g/s which has an error of just 5.4%, compared with the measured value of 74 g/s, [4].

From equation (13), for ψ_o = 27^o, it is obtained k = 0.375.

In the case of the cylindrical bin with D = 300 mm and H = 900 mm, H_1 = 700 mm, using equation (22), for an orifice with D_o = 25.3 mm, it is obtained:

$$t = \frac{2076.1}{K} \text{ (s)} \tag{24}$$

Between those two levels H and H_1 inside the cylindrical bin with a diameter D=300 mm, a quantity of m = 11309 g of seeds is found (obtained through calculations). The flow-time necessary for this quantity, considering a flow like „mass-flow", would be t = 11309/78 ≅ 145 s. Using equation (24) it is obtained K = 14.3. Proceeding the same, the empty-time of the bin, for H = 900 mm, would be t_g = 50893/78 = 652,5 s, and from equation (23) will result t_g = 13971/K, and thus K = 21.4. An average value K_m ≅18 may be estimated, for cylindrical bins.

In the case of the trunk-cone bins at which H = 118 mm, considering H_1 = 100 mm, between the two levels a mass of m = 76 g is found, and so the flow-time of this quantity at the output-level of 78 g/s is: t = 76/78 = 0,97 s. We will determine this time using equation (20).

Using Mathcad Professional 2000 numerical calculation software, for the stated trunk-cone bin data a numerical value of the integral $3.033\cdot10^{-4}$ is to be obtained, and for the term in front of the integral $16.19\cdot10^4$, so that:

$$t = \frac{49.1}{K} \text{ (s)} \tag{25}$$

From equation (25), for a given time t = 0.97 s it is determined K = 51.

Proceeding the same, for the empty-time it is determined t_g = 389.6/78 ≅ 5 s (the volume of the trunk-cone bin is 487 cm^3, and the quantity of seeds is 0.8·487 = 389.6 g). From equation (20) for a given H_1 = 0 it would be determined t_g.

After the numerical calculation of the integral ($1.206\cdot10^{-3}$) is done, it would be determined:

$$t_g = \left(16.19\cdot10^4 / K\right)\cdot 1.206\cdot10^{-3} = 195.25 / K \tag{26}$$

Taking into account that t_g = 5 s, by replacing in (26) it would be determined K = 39. By accepting an average value for K, for the case of the trunk-cone bin, K_m = 45.

The conclusion of the numerical application is that the values of the coefficient K from the reference equation (10) depend, aside the material, on the bin's geometrical shape, which is an interesting figure that needs to be verified by experiments.

Moreover, this numerical calculation offers a procedure to perform experiments in order to determine the values for the coefficient K in different situations.

3. CONCLUSIONS

The flow time of the granular materials through orifices is theoretically grounded in this paper. There are developed complex calculus equations emphasizing the main factors related to the bin geometry and the physical properties of the granular material which influence the flow. The proposed calculus equation allow the elaboration of an experimental procedure for the determination of the coefficient K and, thus, the calculus equation are validated which enables the computers assisted control of the transfer process of the granular materials into the complex technologies flow in working installations

These results are gathered in a paper useful for future research development regarding various aspects of the granular materials flow.

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