A NEW MODEL OF MACRO-ELEMENT USED FOR SHIP HULL TORSION

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Abstract: A proposal of a new model of macro element used to analyse the ship hull torsion – closed section – as thin walled beam using macro elements is treated. The outline of the section is considered as polygonal one. The material is considered as an orthotropic one. For a straight line portion of cross section outline is corresponding a longitudinal strip plate. Due to the warping torsion of the thin walled beam, in the strip plate the stretching-compression, bending and shearing occur. The stiffness matrix of the macro-element is obtained by assembling the stiffness matrices of the strips.In the local strength analysis of the ship hull, the local loading is supposed to be known. These loadings occur from the global strength of the ship hull. An important loading of the ship hull is torsion treated as thin walled beam (frequently as closed section).

Keywords: ship hull torsion, thin-waled beam, FEM analysis

1. INTRODUCTION

In the local strength analysis of the ship hull the local loading is supposed to be known. These loadings occur from the global strength of the ship hull. An important loading of the ship hull is torsion treated as thin walled beam (frequently as closed section). In the following a proposal on the ship hull torsion using macro-elements is presented.

Appropriate behavior of the thin-walled beams with closed cross-section requires no deformation of the cross-section contour of the beam. To satisfy this requirement, used technique is to add transversal stiffening elements.

It was noticed a very strong tendency for contour deformation when the condition of no distortion of the cross-section plane is enforced. In order to eliminate the contour deformation, the stiffening elements have to be very stiff in the transversal plane and the distance between elements to be small.

2. HYPOTHESIS AND CONSEQUENCES

Let us consider a thin-walled beam with closed cross-section under warping torsion load. To each side of the polygonal contour Γ (Figure 1) correspond a strip-plate of constant thickness. Let us denote O as the torsion centre and O^* as the neutral sectorial point. We associate two type reference systems: 1) global reference system $OXYZ$ with the axis OX directed along the longitudinal axis of the beam, 2) local reference system $F_k^0 x_k y_k z_k$ for each stripe-plate , $F_k^0 x_k$ is parallel to OX. We make the following assumptions :

a) The composite material of ship hulls (orthotropic stripe-plates) is linear elastic with the longitudinal modulus of elasticity E in OX direction, the transversal modulus of elasticity G in $F_k^0 y_k z_k$ planes; one direction of orthotropy is OX ,other is $F_k^0 y_k$.

b) The tangential stresses acting in the transversal cross-section of the beam are oriented parallel to the median line Γ and are constant over the entire thickness of the wall.

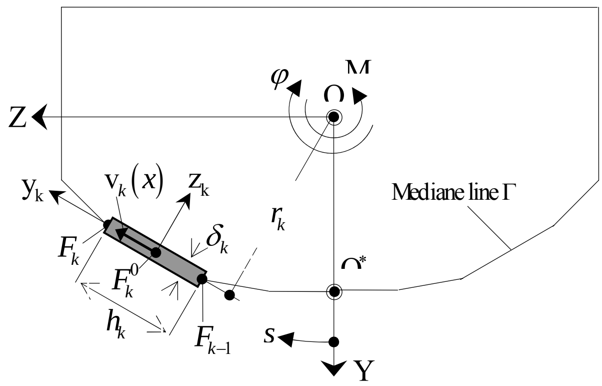

Figure 1

c) The median line Γ of one cross-section is displaced with respect to the crossing plane, but its projection on this plane is unchanged. As aconsequence, for small displacements, the displacement v tangent to Γ is expressed as:

$$v(x,s) = r(s)\varphi(x) \tag{1}$$

where $\varphi(x)$ is the angle of rotation of the cross section and $r(s)$ is the distance from O to the tangent to the median line Γ .

d) The component u of the displacement of points F that is parallel to OX is considered constant over the thickness of the wall but is not necessarily constant over the entire cross-section or over the length of the beam. We assume that u is proportional to the generalized sectorial co-ordinate $\hat{\omega}$ evaluated to O and O^* . Different from classical theory ([1]) or Benscoter theory ([2],[3]) we assume ([4],[5]) that u is proportional to the rate of twist (Figure 2) $u(x,s) = -\hat{\omega}(s)\,\varphi'(x)$ (2)

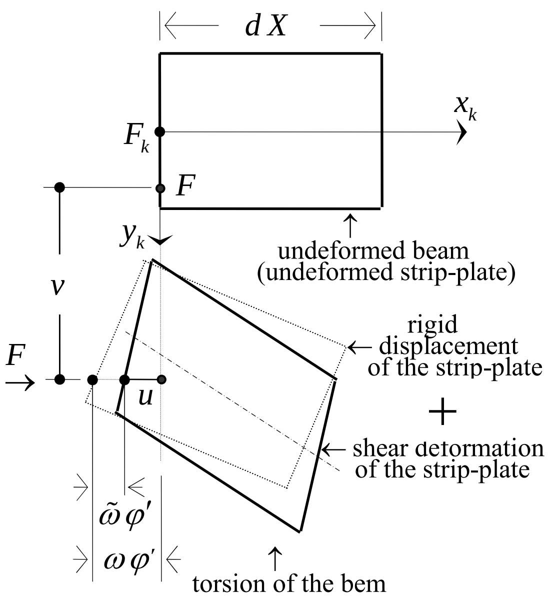

Figure 2

The generalized sectorial co-ordinate is defined as
$\hat{\omega}(s)=\omega(s)-\tilde{\omega}(s)\rightarrow\hat{\omega}_k=\omega_k-\tilde{\omega}_k$ where :

$$\omega(s)=\int_0^s r(s)ds\rightarrow\omega_k=\sum_{i=1}^{k}h_i r_i\,,\tilde{\omega}=\omega_0\tilde{s}/\tilde{S}$$

$$\tilde{\omega}_k=\omega_0\tilde{s}_k/\tilde{S},\omega_0=\oint r(s)ds=\sum_{k=1}^{n}h_k r_k$$

$$\tilde{s}=\int_0^s\frac{ds}{\delta(s)}\rightarrow\tilde{s}_k=\sum_{i=1}^{k}\frac{h_i}{\delta_i}\quad,\quad\tilde{S}=\oint\frac{ds}{\delta(s)}=\sum_{i=1}^{n}\frac{h_i}{\delta_i}$$

where n is the number of strip-plates .

The torsional loading of the beam generates a planar loading of the strip-plate . Using relations (1) end (2) for each strip-plate , one obtains :

$$v_k(x)=r_k\,\varphi(x)\,,\tag{3}$$

$$u_k(x,y_k)=-\hat{\omega}(y_k)\,\varphi'(x)\tag{4}$$

Relations (3) end (4) defines the displacement field for each stripe-plate. The continuity of the displacement u along the jointing edges between two stripe-plates is embedded in relation (4). The linear variation of the generalized sectorial co-ordinate alonthe axis y_k (in the reference system $F_k^0 x_k y_k z_k$ associated to stripe-plate k) may be expressed as

$$\hat{\omega}(y_k)=\hat{\omega}_k^0+(\hat{\omega}_k-\hat{\omega}_{k-1})\eta\tag{5}$$

where $-1/2\le\eta=y_k/h_k\le 1/2$, $\hat{\omega}_k^0,\hat{\omega}_k,\hat{\omega}_{k-1}$ characterize the points F_k^0, F_k, F_{k-1} $\left(\hat{\omega}_k^0=(\hat{\omega}_{k-1}+\hat{\omega}_k)/2\right)$. Using (5) in (4) one obtains $u_k(x,y_k)=-\left[\hat{\omega}_k^0+(\hat{\omega}_k-\hat{\omega}_{k-1})\eta\right]\varphi'(x)$.

Using assumptions **c** and **d** the strain generated in the stripe-plate k will:

$$\varepsilon_k=\frac{\partial u_k}{\partial x}=-\left[\hat{\omega}_k^0+(\hat{\omega}_k-\hat{\omega}_{k-1})\eta\right]\varphi''(x)\ ,\gamma_k=\frac{\partial u_k}{\partial y_k}+\frac{\partial v_k}{\partial x}=\Delta_k\,\varphi'(x)\quad\text{where}\quad\Delta_k=\omega_0/(\tilde{S}\,\delta_k)\ .$$

Normal stresses σ_k appear in each stripe-plate k due to the warping, $\sigma_k(x,y_k)=-E\hat{\omega}(y_k)\varphi''(x)$. These stresses form in each cross-section a system of distributed forces in self-equilibrium.

The tangential stresses τ_k associates with the deformations γ_k , $\tau_k(x)=G\gamma_k=\dfrac{G\omega_0}{\tilde{S}}\dfrac{1}{\delta_k}\varphi'(x)$.The flux of these stresses , $\tau_k\delta_k$, is constant for each section of thin-walled beam.

The differential equation of the twist angle φ obtained by the Ritz method ([4]) is $EI_{\hat{\omega}}\varphi'''-GI_T\varphi'=-M_T(x)$ where $I_{\hat{\omega}}=\sum_{k=1}^{n}I_{\hat{\omega}k}\,,I_{\hat{\omega}k}=h_k\delta_k\left[\left(\hat{\omega}_k^0\right)^2+\left(\hat{\omega}_k-\hat{\omega}_{k-1}\right)^2/12\right],I_T=\omega_0^2/\tilde{S}$,

3. THIN WALLED BEAM MACRO-ELEMENT

Let us consider the thin-walled beam divided in segments (macro-elements) of length L . The macro-elements may by interpret as assembly of n stripe-plates of length L. The assembly enforces the continuity of displacements u along the common edges between two adjacent strip-plates. On consider the following interpolation for the twist angle φ :

$$\varphi(\xi) = h_1(\xi)\varphi_1 + L\,h_3(\xi)\varphi_1' + h_2(\xi)\varphi_2 + L\,h_4(\xi)\varphi_2' \tag{6}$$

where $\xi = x/L$ and : $h_1(\xi) = 1-3\xi^2+2\xi^3$, $h_2(\xi) = 3\xi^2-2\xi^3$, $h_3(\xi) = \xi-2\xi^2+\xi^3$, $h_4(\xi) = -\xi^2+\xi^3$

Let us denote $\boldsymbol{\delta}_\varphi = [\varphi_1, \varphi_1', \varphi_2, \varphi_2']^{\mathrm{T}}$ and $\mathbf{h}(\xi) = [h_1(\xi), Lh_3(\xi), h_2(\xi), Lh_4(\xi)]^{\mathrm{T}}$. Relation (6) becomes

$$\varphi(\xi) = \mathbf{h}(\xi)^{\mathrm{T}}\,\boldsymbol{\delta}_\varphi$$

The continuity of the generalized coordinates φ and φ' in the nodal cross –sections of the macro-element ensures the continuity of the displacements u_k , v_k at all the common edges of adjacent stripe-plates.

Taking into account the assumptions made regarding the loads and strains, one may write the Hook's law in the form

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$$

where $\boldsymbol{\sigma} = [\sigma_k, \tau_k]^{\mathrm{T}}$, $\boldsymbol{\varepsilon} = [\varepsilon_k, \gamma_k]^{\mathrm{T}}$ and

$$\mathbf{D} = \begin{bmatrix} E & 0 \\ 0 & G \end{bmatrix}$$

For the stripe-plate k , the vector $\boldsymbol{\varepsilon}$ becomes $\boldsymbol{\varepsilon}_k = \mathbf{B}(\xi,\eta)_k\,\boldsymbol{\delta}_\varphi$ where

$$B(\xi,\eta)_k = \frac{1}{L}\begin{bmatrix} -\dfrac{1}{L}\left[\hat{\omega}_k^0 + (\hat{\omega}_k - \hat{\omega}_{k-1})\eta\right]\dfrac{\partial^2\mathbf{h}^{\mathrm{T}}}{\partial\xi^2} \\ \Delta_k\,\dfrac{d\,\mathbf{h}^{\mathrm{T}}}{d\xi} \end{bmatrix}$$

The stiffness matrix of the stripe-plate is given by:

$$\mathbf{K}_k = \iiint_{V_k} \mathbf{B}(\xi,\eta)_k^{\mathrm{T}}\,\mathbf{D}\mathbf{B}(\xi,\eta)\,dV_k$$

where V_k is the volume of stripe-plate k . After mathematical manipulations one obtains

$$\mathbf{K}_k = \frac{EI_{\hat{\omega}k}}{L^3}\tilde{\mathbf{K}}_{\mathrm{EB}} + \frac{G}{L}\frac{\omega_0^2}{\tilde{S}^2}\frac{h_k}{\delta_k}\tilde{\mathbf{K}}_{\mathrm{SV}} ,$$

$$\tilde{\mathbf{K}}_{\mathrm{EB}} = \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{symm.} & & & 4L^2 \end{bmatrix} ,\tilde{\mathbf{K}}_{\mathrm{SV}} = \begin{bmatrix} 6/5 & L/10 & -6/5 & L/10 \\ & 2L^2/15 & -L/10 & -L^2/30 \\ & & 6/5 & -L/10 \\ \text{symm.} & & & 2L^2/15 \end{bmatrix}$$

The relation between the generalized forces and generalized displacements has the form $\mathbf{K}\boldsymbol{\delta}_{\varphi}=\mathbf{F}$

$$\text{where } \mathbf{K}=\sum_{k=1}^{n}\mathbf{K}_k=\frac{EI_{\hat{\omega}}}{L^3}\tilde{\mathbf{K}}_{\mathrm{EB}}+\frac{GI_T}{L}\tilde{\mathbf{K}}_{\mathrm{SV}} \tag{7}$$

$\mathbf{F}$ is the loading vector reduced at the nodal cross-sections of the macro-element.

The first term on the right of equation (7) corresponds to stretching/compression end bending (coupled) of the stripe-plates; the stripe-plates behave like Euler –Bernoulli beams. The second term corresponds to Saint - Venant torsion of macro-element.

4. NUMERICAL EXAMPLE

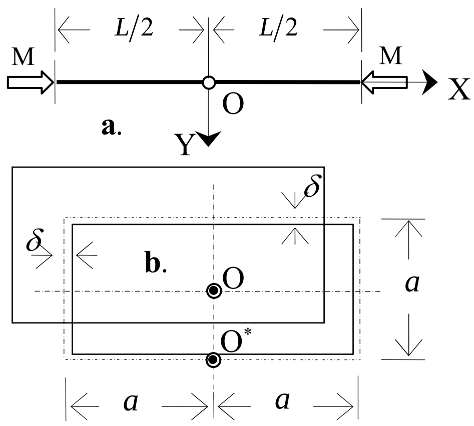

Figure 3

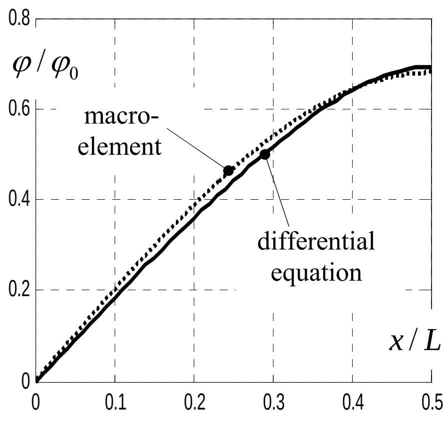

Figure 4

Let us consider a thin-walled of length L (Figure 3-a) with rectangular closed cross-section (Figure 3-b). The beam is made of composite orthotropic material that has $E=15.7\times10^9$ N/m^2, $G=3.4\times10^9$ N/m^2 . The distortion of the cross-section plane is prevented in sections $x = \pm L/2$. The beam has loading torques M at the ends. The following numerical values were used :

$$a = 5\times10^{-2}\,\mathrm{m}\,,\delta = 3\times10^{-3}\,\mathrm{m}\,,(\,I_T = 10^{-6}\,\mathrm{m}^4\,,I_{\hat{\omega}} = 5.2\times10^{-11}\,\mathrm{m}^6\,)$$

The thin walled beam is considered single macro-element of length $L=2a$. In Figure 4 we reveal the variation of ratios φ/φ_0 (φ_0 =rotation of the ends to Saint Venant torsion).

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